Signal Processing for Medical Applications –

Frequency Domain Analyses

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Finite element method (FEM)

- The FEM has a additional advantage that it can capture anisotropic conductivities of the domain being modelled.

- The main idea behind the FEM is to reduce a continuous problem with infinitely many unknowns field values to a finite number of unknowns by discretizing the solution region into elements.

- The value at any point in the field can then be approximated by interpolation functions within the elements.

- These interpolation functions are specified in terms of the field values at the corners of the elements, points known as nodes.

- It is to be noted that for linear interpolation potentials, the electric field is constant within an element.

Finite element method (FEM)

- Given a geometric model, the FEM proceeds by assembling the matrix equations to build the stiffness matrix A.

- Boundary conditions are then imposed and source currents are applied. These boundary conditions and source conditions are incorporated within the vector b .

- Application of the FEM reduces Poisson's equation to the linear system

$$A_{ij}\phi_j = b_i \tag{29}$$

where ϕ are the unknown potentials at the nodes of the volume.

- The traditional method of constructing the L_e matrix is to place three orthogonal sources in each cell of a volume domain, and for each dipole source, compute the voltages at the electrodes.

- For a volume mesh consisting of N tetrahedral elements, this requires computing $(N \times 3)$ forward solution.

Finite element method (FEM)

- The two methods for constructing the lead field matrix L_f .

Element Basis:

- The constraints here are to achieve the maximal possible resolution of sources for the model: one dipole per tetrahedral element.

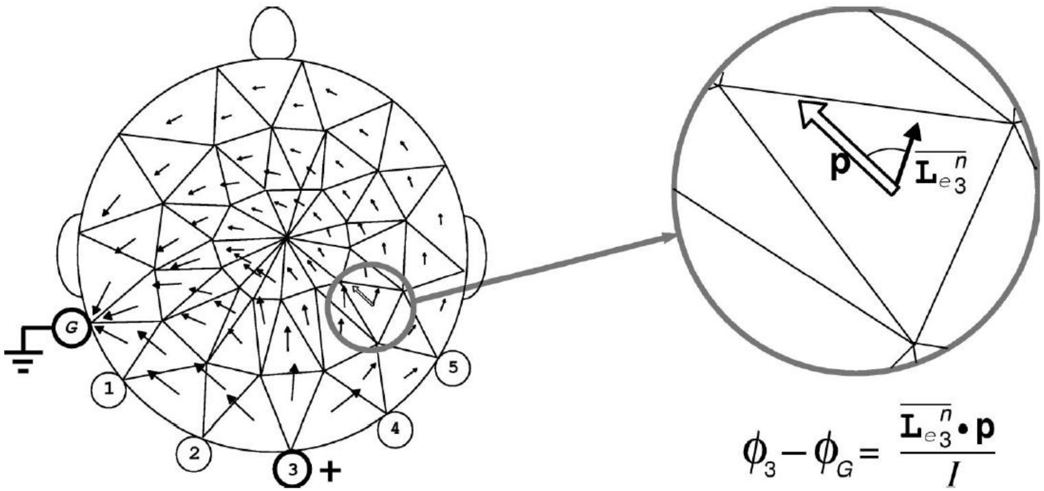
- We compute the potentials not only on the surfaces (as in BEM), but through the entire volume.

- Both the goals can be achieved by using the principle of reciprocity- applicability of reciprocity to anisotropic conductors.

- It stated that given a dipole (an equivalent source), p, and a need to know the resulting potential difference between two points A and B, it is sufficient to know the electric field E at the dipole location resulting from a current, I, placed between points A and B:

$$\frac{(E \cdot P)}{-I} = \phi_A - \phi_B \qquad (30)$$

Finite element method (FEM)



$$\phi_3 - \phi_G = \frac{\overline{\mathbf{L}_{e3}^{n}} \bullet \mathbf{p}}{I}$$

- The depiction of the reciprocity-based method. A unit current is applied between electrodes 3 and G. The reciprocity principle states that the voltage difference between 3 and G due to a dipole source p placed in element e will be equal to the dot product of p and the electric field e .
- So, rather than iteratively placing a source in every element and computing a forward solution at the electrodes we can ‚invert' this process: we place a source and sink at pairs of electrodes, and for each pair compute the resulting electric field in all of the elements.

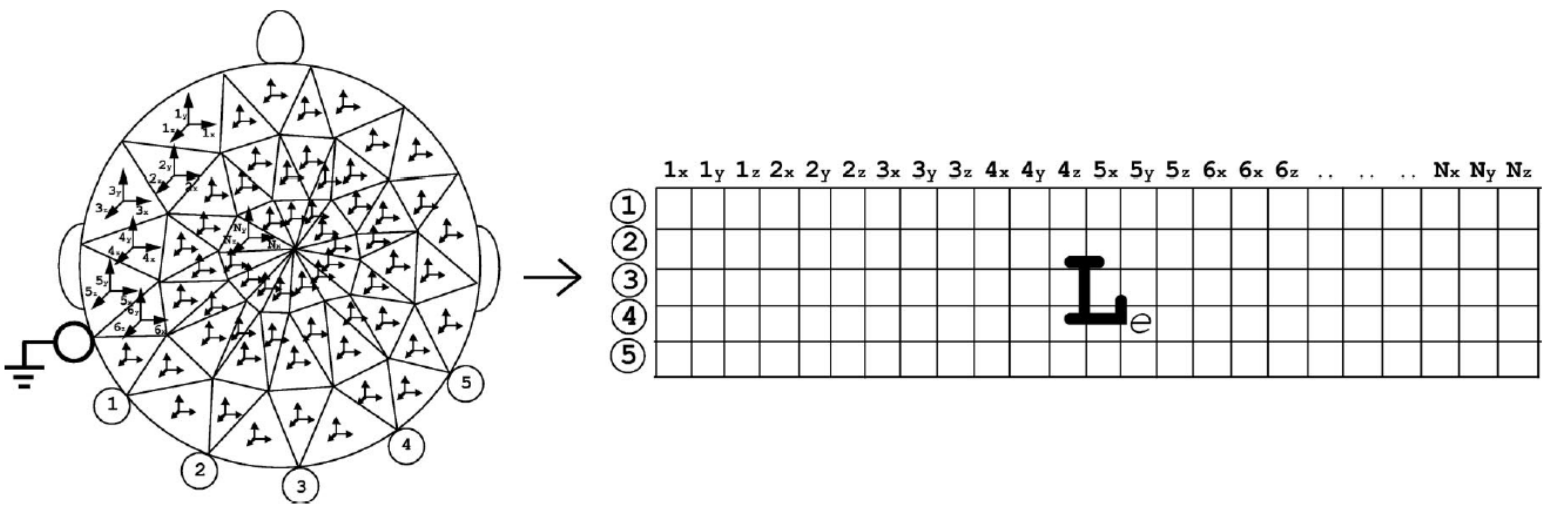
Finite element method (FEM)

- By using the reciprocity principle to reconstruct the potential differences at the electrodes for a source placed in any element.
- The construction proceeds as follows: First we choose one electrode as ground (i.e., by forcing ist potential to zero).
- For each of the other M electrodes, one at a time, we place a current source, I, perpendicular to the surface at that electrode and a unit current sink at the ground electrode.
- The forward solution is then computed, resulting in a potential field, ϕ, defined at each node in the domain.
- We take the gradient of this potential field, yielding electric field, E, at each element in the head.

Finite element method (FEM)

- A row of the lead field L_e is computed by evaluating $(E-I)$ in every element. This process is repeated for each of the M source electrodes, producing the L_e matrix satisfying

$$L_e s_e = \phi_r \qquad (31)$$



- The depiction of the element-oriented lead-field basis. Each orthogonal dipole in each element corresponds to a column of L, and each electrode corresponds to a row of L. Each entry of L corresponds to the potential measured at a particular electrode due to a particular source.

Finite element method (FEM)

Node Basis:

- The method for deriving the element-oriented lead field constructs an L_e basis that maps dipole components placed at elements to potentials at the scalp-recording electrodes.
- The alternative formulation is based on the divergence of the source current density vector at each node, rather than three orthogonal current dipoles within each element.
- The node-oriented basis is derived directly from the finite element stiffness matrix, A, and the right-hand side vector, s_n .
- It is straight forward to solve the well-conditioned system

$$\phi = A^{-1} s_n \tag{32}$$

to recover the potentials, ϕ, throughout the volume when the sources are known.

Finite element method (FEM)

- For source imaging, however we are interested not in the potentials everywhere in the volume, but only in the potentials at those few nodes corresponding to scalp electrodes recording sites.

- In this case a matrix R is introduced that selects just the electrode potentials from ϕ . R is a $[K \times M]$ matrix (number of nodes by less that the number of recordings electrodes).

- Each row of R contains a single non-zero entry: the value 1.0 located at the column corresponding to the node index for that electrode.

- From equation (32), we now select a subset of ϕ by applying R:

$$\phi_r = R\phi = RA^{-1}s_n \tag{33}$$

- The RA^{-1} operator is a node-oriented lead-field basis, which we term L_n, and for it follows that:

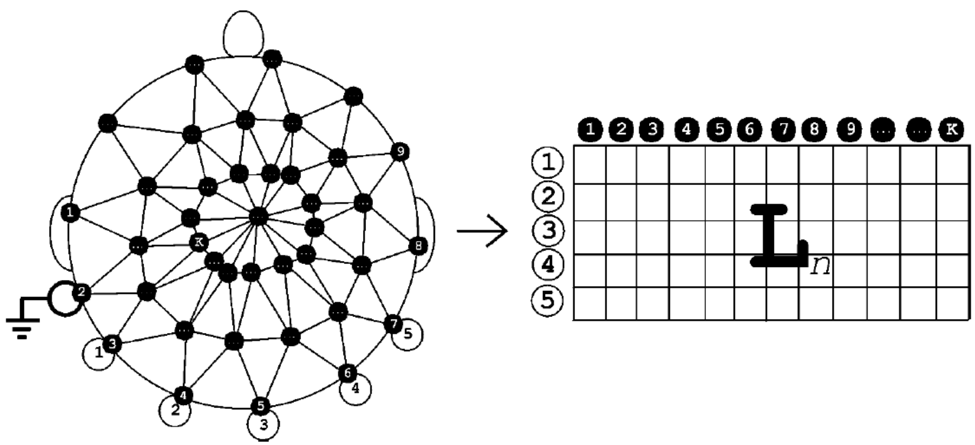
$$L_n s_n = \phi_r \tag{34}$$

Finite element method (FEM)

- Inorder to efficiently compute RA^{-1}, we can exploit the sparse nature of R. Since R contains only M nonzero entries, we need to construct only the corresponding M columns of A^{-1}. This is accomplished by solving the equation

$$A(A^{-1})_m = I_m \qquad (35)$$

where $(A^{-1})_m$ is unknown for source m. As with the construction of the L_e basis, this technique requires generating M forward solutions.



- In contrast to the L_e, this matrix column corresponds to orhthogonal dipoles, the columns now corresponds to nodes. It has approx. 94% fewer columns and best suited for distributed source configurations

Finite element method (FEM)

- The two lead fields, element oriented and node oriented differ in several relevant ways:
- The L_e formulation is based on having a dipole moment of a particular strength and orientation in each element.
- L_e is more useful for reconstructing discrete dipolar sources. This is an appropriate method for localizing very focal neural activity, such as epileptic seizures or specific motor control tasks.
- In contrast, the node-oriented L_n lead field is defined with the values at the nodes. This means L_n will work best for recovering less focal, more distributed-type sources which are characterized by coordinated activity occuring at multiple neural locations.
- Such a solution should be well-suited to capture diffuse cognitive events, such as language processing or the performance of complex tasks.

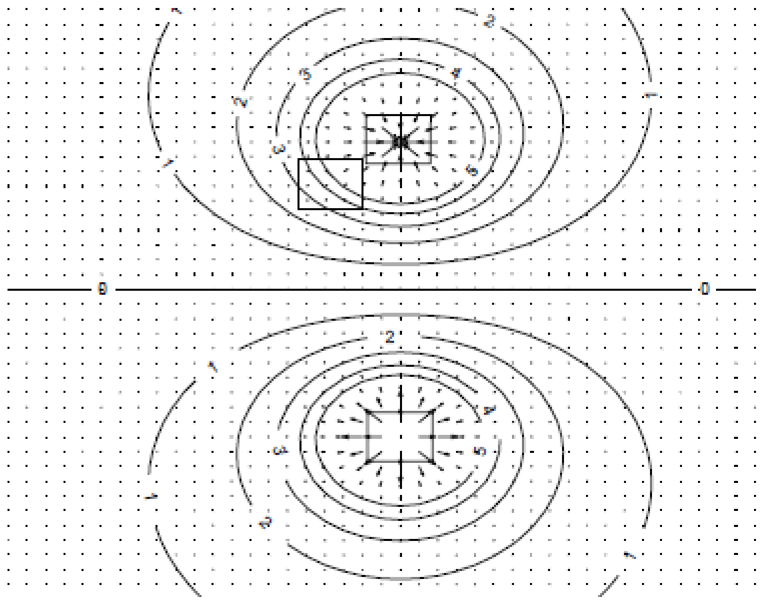
Finite element method (FEM)

- The size of L_e basis is $[M \times (N \times 3)]$, one less than the number of recording electrodes by three times the number of the elements. Million elements in a finite element mesh corresponds to $M = 64, 128, 256$ recording electrodes.

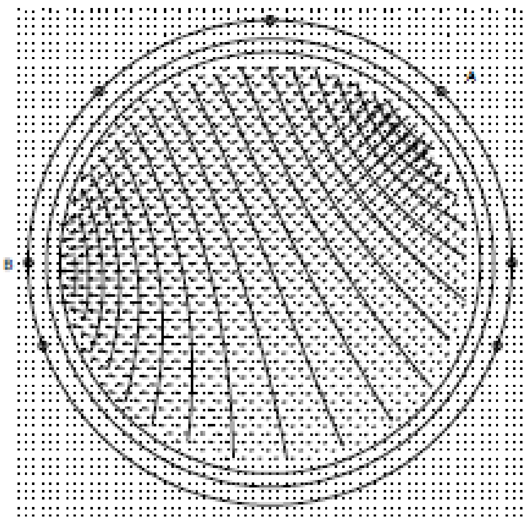
- By using this lead-field basis for source imaging, it is clear that the solution will be grossly under-determined.

- The node-oriented basis, L_n, is somewhat smaller $[M \times K]$, one less than the number of recording sites by the number of nodes. There are still typically many more nodes as hundred thousand than electrodes, the system is less under-determined than the element based formualtion.

Finite element method (FEM)

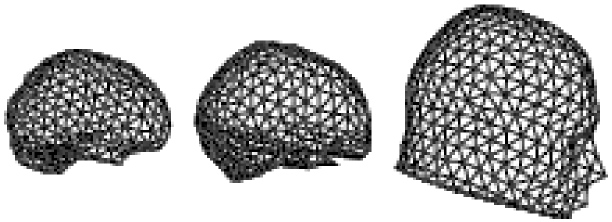


The current density and equipotential lines in the vicinity of a dipole. Current source current sink is given. Boxes are illustrated which represents the volume.

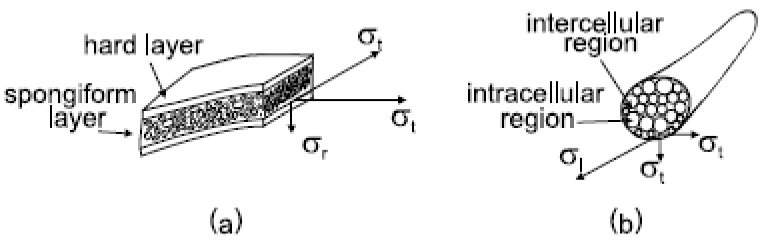


Lead field between two electrodes. The current density and the equpotential lines are illustrated when introducing a current at electrode A and removing the same amount at electrode B.

Finite element method (FEM)



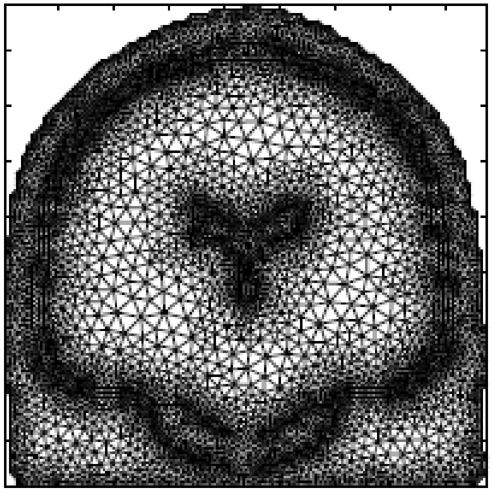
Example mesh of the human head used in BEM. Traingulated surfaces of the brain, skull and scalp compartment used in BEM. The surfaces indicate the difference interfaces of the human head: air-scalp scalp-skull and skull-brain.



Aniotropic conductivity of the brain tissues.

a) The skull consists of 3 layers: a spongiform layer between two hard layers. The conductivity tangentially to the skull surface is 10 times larger than the radial conductivity.

b) White matter consist of axons, grouped in bundels. The conductivity along the nerve in the bundles is 9 times larger than perpendicular to the nerve bundle.



Example mesh in 2D used in FEM. A digitization of the 2D coronal slice of the head. The 2D elements are the traingles.

Topics of Presentation

Topics	Student Name
1) Signal processing is MEG	
2) Mapping the SNR of cortical sources in MEG/EEG	Ali Alfaraoon – 18/25-01-2013
3) Comparison of EEG and MEG in source level	Masoud Sarabi – 18/25-01-2013
4) FEM for forward Modelling	
5) Sparse source imaging	Jayjit Dutta – 18/25-01-2013
6) Eigenspace projection beamformers	Roos Pascal – 18/25-01-2013
7) MEG/EEG source reconstruction using NUTMEG	Sven Jaschke – 25/01-02-2013
8) Mapping human brain with MEG and EEG	Julius Schmalz - 25/01-02-2013
9) Data driven time frequency analysis	Sumit Jha -25/01-02-2013
10) Power envelope correlations – source analysis	Mushfa Yousuf – 25/01-02-2013

Topics of Presentation

Topics	Student Name
11) Overview on artifact correction algorithms – Gradient	Necati Ugras Babacan – 01/08-02-2013
12) Overview on artifact correction analysis – BCG artifact	
13) Spatial-temporal signal separation method	Andre Iwers – 01/08-02-2013
14) Phase amplitude coupling between neuronal oscillations of different frequencies	Sami Alkubti Almasri – 01/08-02-2013
15) Driver Fatigue: EEG and pschological assessment	Stephan Senkbeil – 01/08-02-2013

Topics of Presentation

Topics	Student Name
16) Review on directionality methods	Riya Paul – 08/15-02-2013
17) Review of brain connectivity in EEG/MEG	Sandra Schmidt – 08/15-02-2013
18) Resting state FMRI	Thi thu Hien Vu – 08/15-02-2013
19) New and emerging techniques for brain mapping	Balachandar Vittal – 08/15-02-2013
20) Analyzing effective connectivity in FMRI	Sönke Heidkamp and Christin Baasch -08 /15-02-2013
21) NIRS development and field of application	Marco Klein – 08/15-02-2013

Topics of Presentation

Time Slots	Dates of Presentation
9:15 – 9:30	25-01-2013
9:35 – 9:50	01-02-2013
9:55 – 10:10	08-02-2013
10:15 – 10:30	
9:15 – 9:30	
9:35 – 9:50	
9:55 – 10:10	15-02-2013
10:15 – 10:30	
10:35 – 11:05	
11:10 – 11:25	

Grading system

Paper: 50 %

- **Individual Intiative**
- **Understanding the subject**
- **Writing Skills**

Presentation : 50 %

- **Timing**
- **Effective answering**
- **Attendance**