



# Signal Processing for Medical Applications – Frequency Domain Analyses

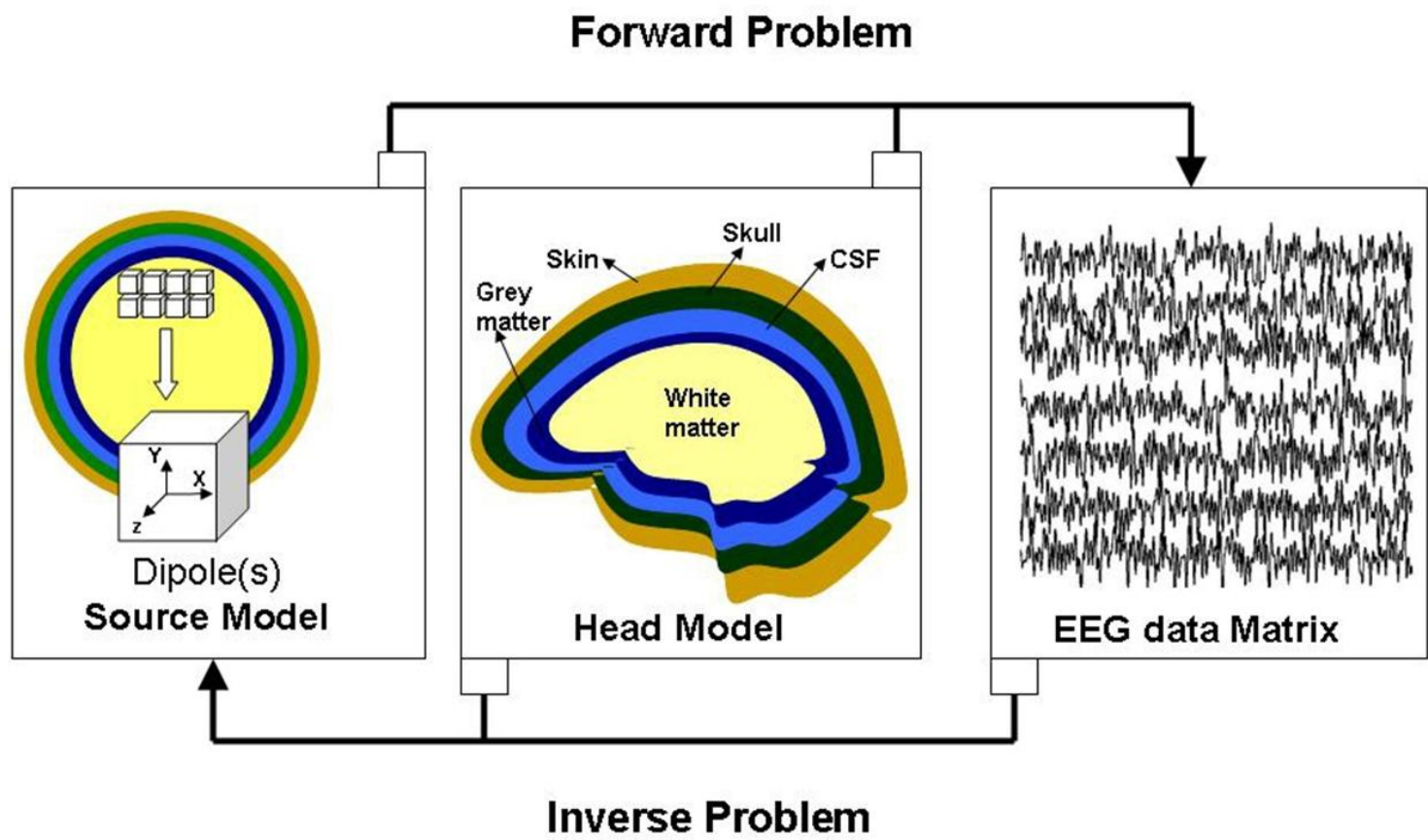
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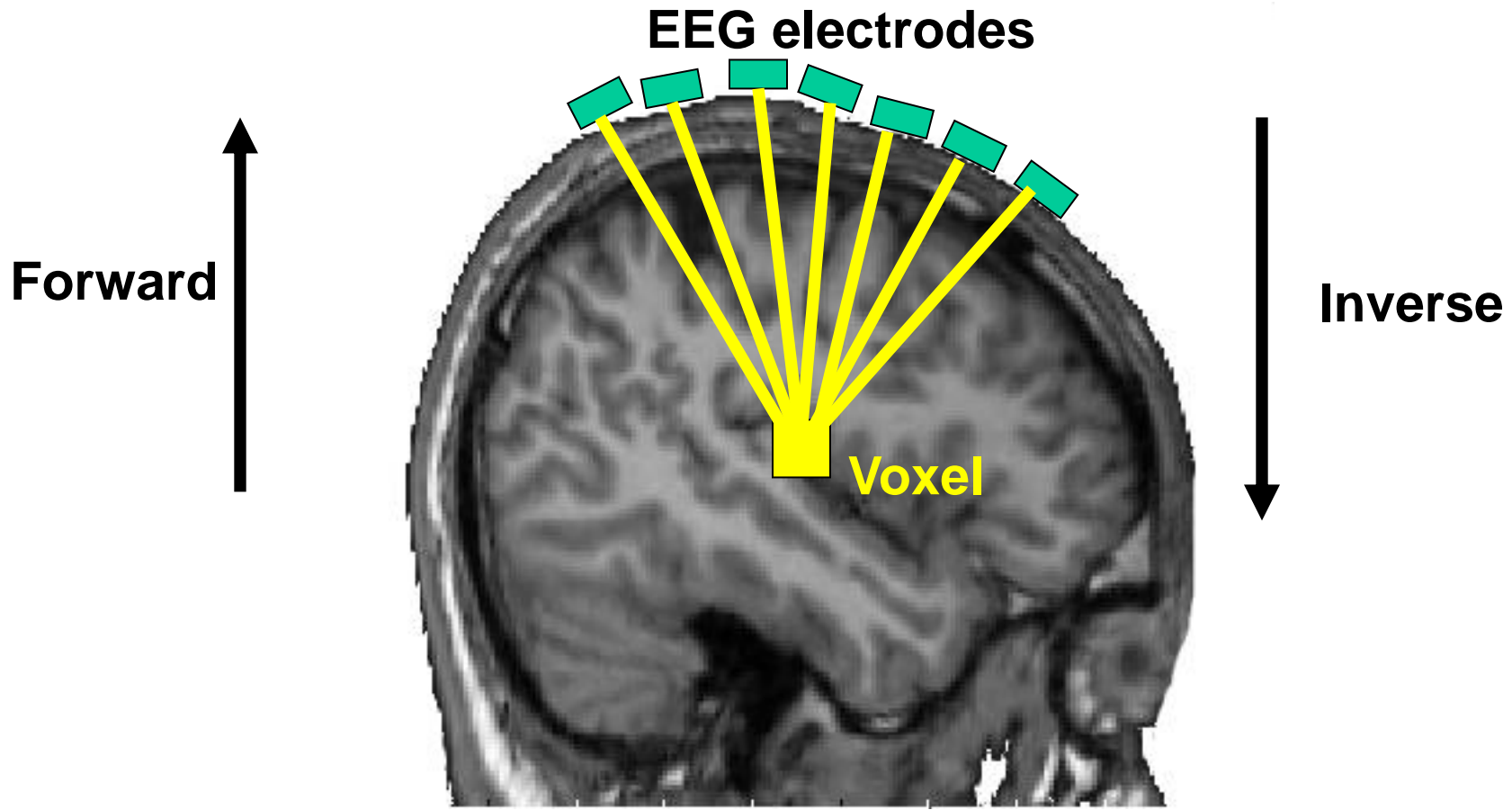


## Forward problem

- Constructing a volume conduction model for the head using the electrode locations



## Beamforming approach



## Forward solution

- In order to determine the transmission from sources in the brain to the surface of the head where the EEG electrodes are placed, a ***volume conduction model*** is used with a ***boundary-element method***.
- Two models of the brain: Single sphere and five concentric spheres model.
- The head is modeled by giving in the radius and the position of the sphere with the electrode locations.
- In order to map the current dipoles in the human brain to the voltages on the scalp the lead-field matrix needs to be calculated.

## Lead Field Matrix

- The distribution of an electromagnetic field in the head is described by the linear Poisson equation as:

$$\Delta \cdot (\sigma \Delta \phi) = \Delta \cdot J^s \text{ (in } \Omega) \quad (6.1)$$

where  $\sigma$  is the electrical conductivity,  $\phi$  is the electrical potential,  $J^s$  is the electric current at source position  $s$ , giving the distribution of the electromagnetic field in the head.

- From the linearity of the equation (22)  $\Rightarrow$  follows that the mapping from electric sources within the brain to the scalp recordings  $\Rightarrow$  outside of the scalp  $\Rightarrow$  represented by a linear operator  $L_f$ :

$$\phi_r = L_f s + n \quad (6.2)$$

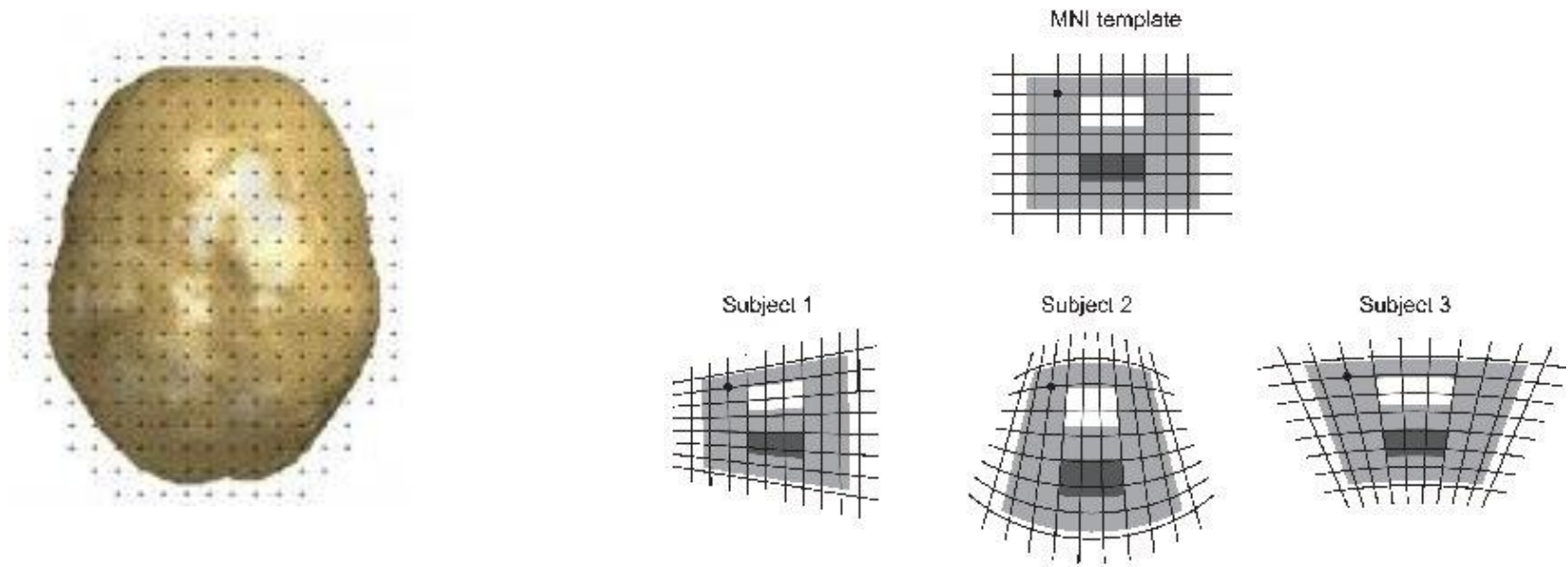
where  $\phi_r$  is the resultant recordings,  $n$  is the noise and  $L_f$  is the lead field matrix.

- In this model the lead-field matrix,  $L_f$ , contains the information about the geometry and conductivity of the model.

# Lecture 6 – Source analysis in frequency domain

## Lead Field Matrix

- The lead field matrix  $L_f$  defines a projection from current sources at discrete locations in the brain to potential measurements at discrete recording sites on the scalp.
- $L_{fij}$  → to potential  $\phi_i$  → measured at recording site  $j$  → due specifically to source  $S_j$ .
- Sources are defined by three orthogonal dipoles,  $S_{jx}, S_{jy}, S_{jz}$  and source positions are generally located on a regular grid of rectangular cells covering the domain of interest.

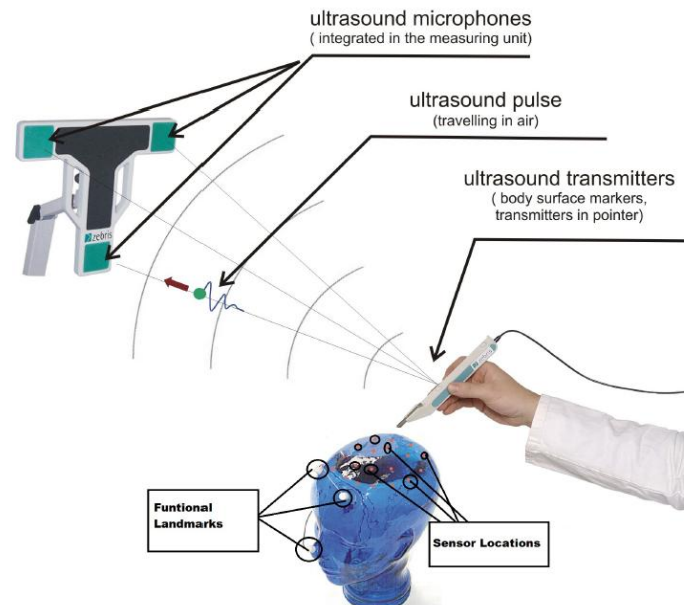


## Lead Field Matrix

- The coordinates of the electrodes and the dipoles are needed in order to be able to compute  $L_f$ .
- The coordinates of the dipoles were derived in the following way: A rectangular grid of current dipoles which were placed on three-dimensional cubes called voxels (volume pixels) was applied.
- The voxel coordinates applied were derived by considering a rectangular grid of  $27 \times 23 \times 27$  voxels with distance 5mm.
- A average human brain was then laid over these voxels and the 8723 voxels which were covering this brain were marked and used for the source analysis.

## Lead Field Matrix

- The coordinates of the electrodes can be obtained using the Zebris system below:



- The matrix  $L_f$  can be generated using the boundary element method (BEM) for the spherical models.

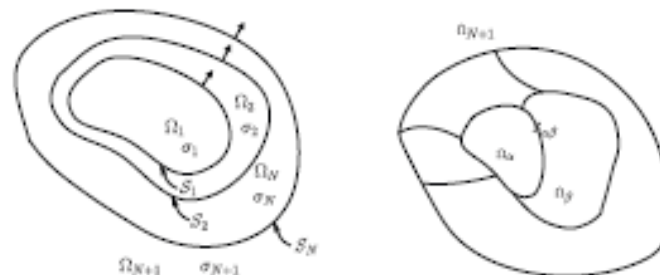


## Conductivity Model

- The choice of the conductivity model is very delicate because it strongly constraints the numerical solutions that can be used to solve the problem.

isotropic - Having a physical property that has the same value when measured in different directions – anisotropic.

- The conductivity field  $\sigma$  should be modeled as a tensor field, because composite tissues such as bone and fibrous tissues have a effective conductivity that is anisotropic.
- Realistic, anisotropic conductivity models can however be difficult to calibrate and handle: simpler, semi-realistic, conductivity models assign a different constant conductivity to each tissue.



## Conductivity Model

- There are three main types of conductivity models, and associated numerical methods:
  - If the conductivity field can be described using simple geometries (with axilinear, planar, cylindrical, spherical or ellipsoidal symmetry), analytical methods can be derived, and fast algorithms have been proposed that converge to the analytical solutions for EEG and MEG.
  - For piecewise constant conductivity fields, boundary element methods (BEM) can be applied, resulting in a simplified description of the geometry only on the boundaries.
  - General non-homogenous and anisotropic conductivity fields are handled by volumic methods; Finite element methods and finite difference methods belongs to this category.

## Overview of the Conductivity Models

- The accuracy of the forward solvers can be assessed for simple geometries such as nested spheres, by comparison with analytical results.
- The precision of the forward solution is tested with two measures, the RDM (relative difference measure) and the MAG (magnitude ratio).
- The RDM between the forward field given by a numerical solver  $g_n$  and the analytical solution  $g_a$  is defined as:

$$RDM(g_n, g_a) = \left\| \frac{g_n}{\|g_n\|} - \frac{g_a}{\|g_a\|} \right\| \in [0, 2] \quad (6.3)$$

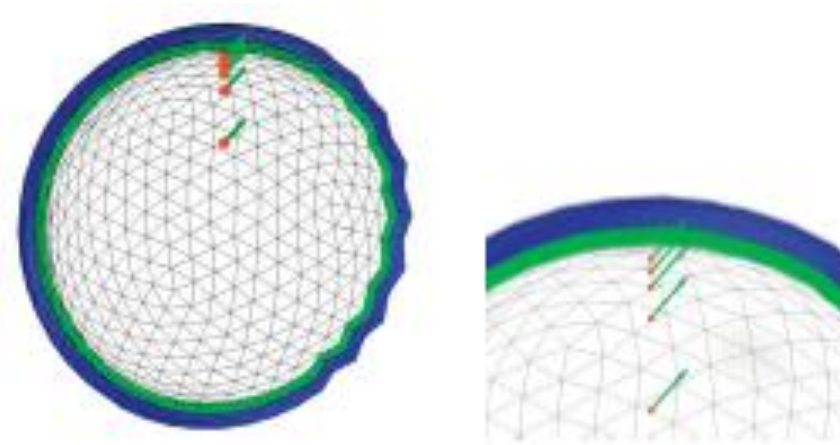
while the MAG between the two forward fields is defined as:

$$MAG(g_n, g_a) = \frac{\|g_n\|}{\|g_a\|} \quad (6.4)$$

In both of these expressions, the norm is the discrete  $l^2$  norm over the set of sensor Measurement. The closer to 0 (resp. to 1) the RDM (resp. the MAG), the better it is.

## Geometrical Models

- The comparisons were made both on classic regular sphere meshes as shown below and on random meshes.



- A random sphere mesh with unit radius and  $N$  vertices is obtained by randomly sampling  $10N$  3D points, normalizing them, meshing them, meshing their convex hull and decimating the obtained triangular mesh from  $10N$  to  $N$  vertices. This process guarantees an irregular meshing while avoiding flat triangles.
- The radii of the 3 layers are set to 88, 92 and 100, while the conductivities of the 3 homogeneous volumes are set to 1,  $1/80$ (skull) and 1.

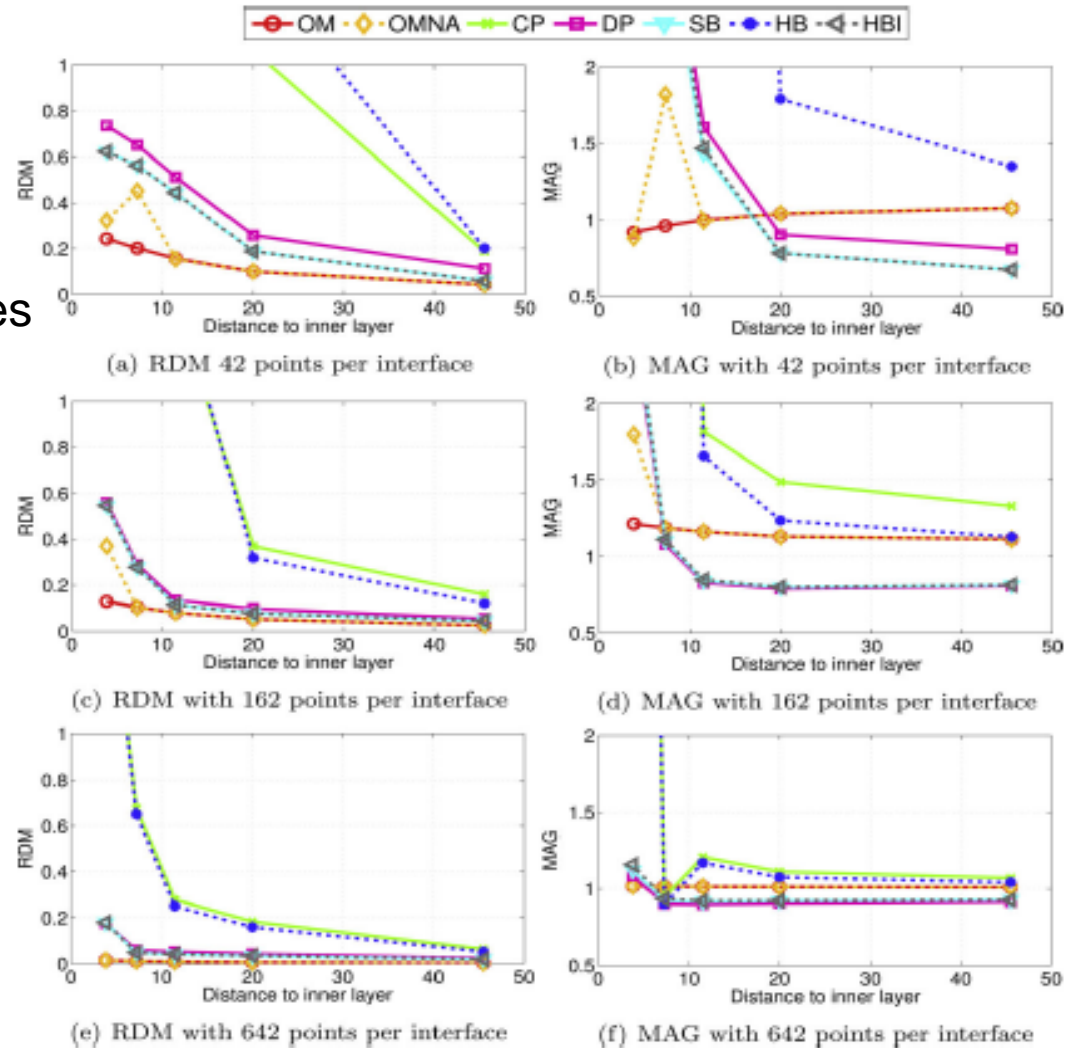
## Geometrical Models

- The BEM solvers are tested with three nested spheres which model the inner and the outer skull, and the skin. For each randomly generated head model, it was tested that they were no intersection between each mesh.
- The results with regular spheres are presented here for these available models
  - OpenMEEG uses symmetric BEM – with and without adaptive integration (OM and OMNA).
  - Simbio (SB)
  - Helsinki BEM with and without ISA(isolated skull approach) (HBI and HB)
  - Dipoli (DP) – simple linear collocation method
  - BEMCP (CP) – simple linear collocation method

## Geometrical Models

Accuracy comparison of the different BEM solvers for EEG:

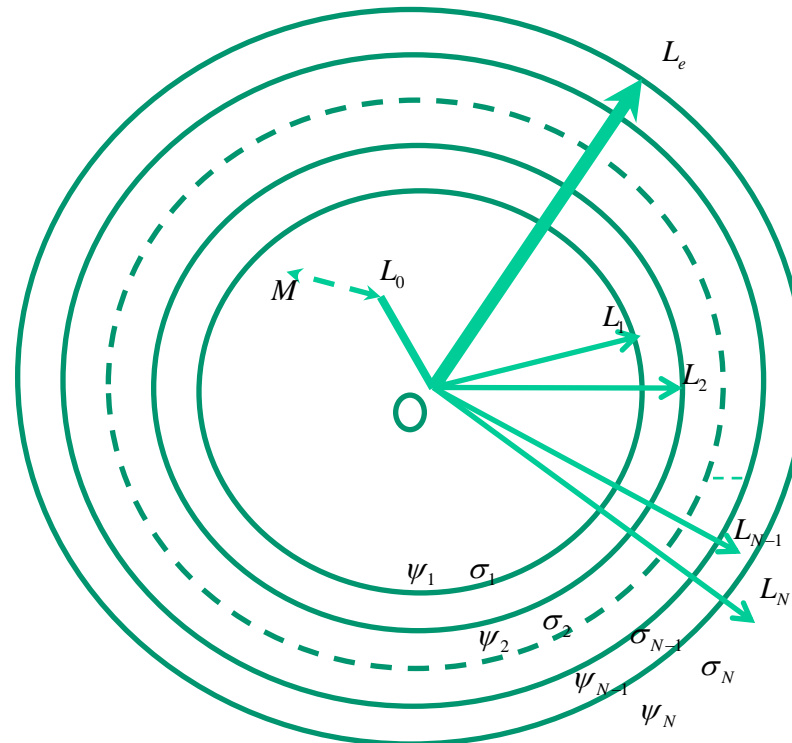
- 42 vertices per 42 EEG electrodes
- 162 vertices per 162 EEG
- 642 vertices per 642 EEG



## Boundary element method (BEM)

- The BEM allows to calculate the electric potential  $\phi$  of a current source in an inhomogeneous conductor by solving the following integral equation:

$$\bar{\sigma}_k \phi(r) = \sigma_0 \phi_0(r) + \frac{1}{4\pi} \sum_{i=1}^{n_s} \Delta\sigma_i \oint_{S_i} \sigma(nr') \cdot \frac{r' - r}{|r' - r|^3} dS'_i \quad (6.5)$$



## Boundary element method (BEM)

- If the conducting object is divided by closed surfaces  $S_i (i=1,2,\dots,n_s)$  into  $n_s$  compartments, each having a different enclosed conductivity  $\sigma_j^{in}$ , the electric current due to electric potential at position  $r \in S_k$  is then given in equation (6.5).
- $\phi_0$  is representing the potential of the source in an unlimited homogeneous medium with conductivity  $\sigma_0$ , the mean conductivity is  $\sigma_k = (\sigma_k^{in} + \sigma_k^{out})/2$ , where  $\sigma_k^{out}$  is the unenclosed isotropic conductivity and the conductivity differences are given as  $\lambda\sigma_i = \sigma_i^{in} - \sigma_i^{out}$ .
- Estimating electric field  $\rightarrow$  approximate numerically the integral over the closed surface  $S_i$  of the conductor boundaries consisting of differential surface elements ( $dS'_i$ ) and with surface normal orientations  $n$  at positions  $r'$ .



## Boundary element method (BEM)

- The potential values of the basis functions form a vector of unknowns which is used to approximate the potentials on the surface elements, which can be solved as follows:

$$\bar{\sigma}\phi = \sigma_0\phi_0 + B\phi \Rightarrow \phi = (\sigma - B)^{-1} \sigma_0\phi_0 \quad (6.6)$$

- If the equation (6.5) solves (6.6) just for the fixed number of measurements positions a transfer matrix  $T$  is obtained, that relates the sensor signals to the homogeneous potentials.
- The potential vector  $\phi$ , that contains the field distribution at all skin nodes, generated by a (dipolar) source inside the innermost compartment (brain) can thus be easily computed by a simple matrix vector multiplication:

$$\phi = T\phi_0 \quad (6.7)$$

with

$$T = (\sigma - B)^{-1} \sigma_0 \quad (6.8)$$

## Boundary element method (BEM)

- The column vector  $\phi_0$  contains the electric potential values  $\phi_{0i}$  of all skin-nodes  $i$  at position  $r_i$  for the source in an infinite homogeneous conductor with conductivity  $\sigma_0$  (dipole at position  $r_j$ ; current  $j$ ):

$$\phi_{0i} = \frac{1}{4\pi\sigma_0} j \frac{r_i - r_j}{|r_i - r_j|^3} \tag{6.9}$$

- The BEM generalized model for  $i = 1, 2, \dots, n_s$  explained here was used for the construction of a simple single-sphere model for which the parameter  $S_i$  described in equation (6.5) will be  $S_i = i = 1$ , and the complex five concentric-spheres model were  $S_i = i = 5$  for the reconstruction of the sources.

