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Signal Processing for Medical Applications –

Frequency Domain Analyses

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Phase

- The phase can be estimated from the argument of the cross-spectrum

$$\phi(\omega) = \arg\{S_{xy}(\omega)\} \tag{5.1}$$

- The phase can be reliably estimated only in the frequency band of the significant coherence, the phase estimate $\phi(\omega)$ and its upper and lower 95% confidence interval are given by

$$C_L = \phi(\omega) \pm 1.96 \left[\frac{1}{2M} \left(\frac{1}{C(\omega)} - 1 \right) \right]^{\frac{1}{2}} \tag{5.2}$$

- Thus, the confidence interval of the phase estimate is inversely related to coherence.

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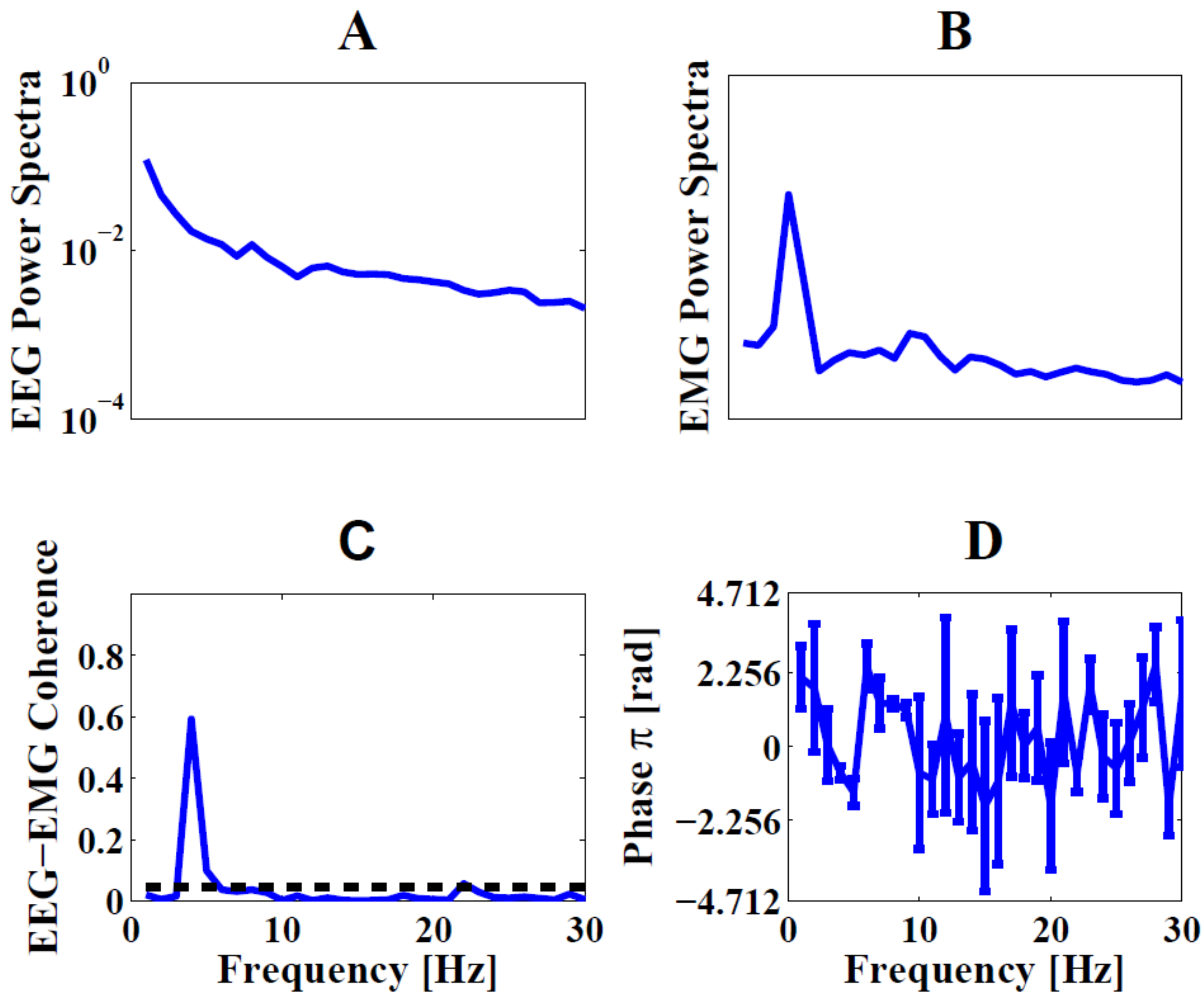
Spectrum



Different ways of estimation of Delay

- Four different ways of estimating delay in biomedical signals

  - Maximum of cross correlation
  - Pointwise interpretation of phase spectrum
  - Fitting a straight line to the phase spectrum
  - Hilbert transform Method

Maximum of cross correlation:

If $y(t)$ is a time-shifted copy of $x(t)$, i.e., $y(t+\delta) = x(t)$, the delay δ can be estimated as the lag at which the estimated cross correlation between input and output is maximal:

$$\hat{\delta} = \max_{\tau} \left| CC_{xy}(\tau) \right|. \quad (5.3)$$

Different ways of estimation of Delay

Pointwise interpretation of the phase spectrum:

This method is based on the phase spectrum between input and output. In general, for every linear system

$$y(t) = \int_{-\infty}^{\infty} d\tau \, a_{xy}(\tau)\, x(t-\tau) = (a_{xy} * x)(t) \tag{5.4}$$

the system's impulse response function $a_{xy}(\tau)$ can be written as the convolution of three subsystems with specific properties:

$$a_{xy}(\tau) = (delay_{\delta} * mp * ap)(\tau) \tag{5.5}$$

where the first component is a pure delay, mp is so called minimal phase system, and ap is an all-pass filter.

The phase spectrum $\phi_{xy}(f)$ is determined solely by the function a_{xy}, since by Eq. (5.4),

$$S_{xy}(f) = \tilde{a}^{*}_{xy}(f)\, S_x(f), \tag{5.6}$$

Different ways of estimation of Delay

If $S_x(f)$ is real,

$$\phi_{xy}(f) = \arg S_{xy}(f) = \arg \tilde{a}^*_{xy}(f) \qquad (5.7)$$

If the system consists of a delay only, $a_{xy}(\tau)$ is a delta distribution at lag δ. The fourier transform of this transfer function is

$$\tilde{a}_{xy}(f) = \exp(-2\pi i f\delta) \qquad (5.8)$$

From which it follows that the phase spectrum is a straight line through the origin with the slope given by the delay time δ :

$$\phi_{xy}(f) = 2\pi f\delta \qquad (5.9)$$

In the point wise interpretation method, one assumes a delay-only model and uses the above equation at a single frequency to infer the delay time between input and output.

The frequency at which the coherence is maximal will yield the smallest error for the delay time estimate.

$$\hat{\delta} = \hat{\phi}_{xy}(f_c) / 2\pi f_c, \quad \text{where} \quad f_c = \max_f coh_{xy}(f) \qquad (5.10)$$

Different ways of estimation of Delay

Fitting a straight line to the phase spectrum:

A point-wise interpretation of the phase spectrum is certainly suboptimal even if the delay-only model holds, since it ignores the information present in the rest of the phase spectrum.

This informaton can be used by fitting a straight line to a part of the phase spectrum that shows significant coherency.

Generic least-squares or weighted least-squares is hampered by the fact that the phase estimate is only defined up to a modulus of 2π - continous phase spectrum – low signal to noise ratio.

The fit can be achieved by a cosine function which is locally quadratic and automatically takes care of the periodicity problem. The weights are chosen to reflect the variance of the phase spectrum estimate.

Different ways of estimation of Delay

Thus, one defines the objective function

$$obj(\delta) = \sum_{f_j \in B} \frac{coh^2(f_j)}{1 - coh^2(f_j)} \cos\left[\phi_{xy}(f_j) - 2\pi\, f_j \delta\right] \qquad (5.11)$$

where B is either the full range of available frequencies with significant coherency or a subset. This makes it possible to restrict the fitting to physiologically important frequencies for a given application.

The delay time is given by the delay at which the objective function is maximal:

$$\delta = \max_{\delta} obj(\delta) \qquad (5.12)$$

Different ways of estimation of Delay

Hilbert Transform Method:

The improvement of the previously described methods can be achieved by correcting the phase spectrum before fitting a straight line to it.

If one could subtract the contributions of the minium phase system and the all-pass system, the delay-only model would hold, and the fitting proedure described before could be applied to linear system without any constraints.

In many physical systems it can be modeled as minimum phase systems without an all-pass component, e.g., as AR models.

The objective function is defined as referring to Equ. (5.5):

$$obj(\delta) = \sum_{f_j \in B} \frac{coh^2(f_j)}{1 - coh^2(f_j)} \cos\left[\phi_{xy}(f_j) - \arg mp_{xy}(f_j) - 2\pi f_j \delta\right] \quad (5.13)$$

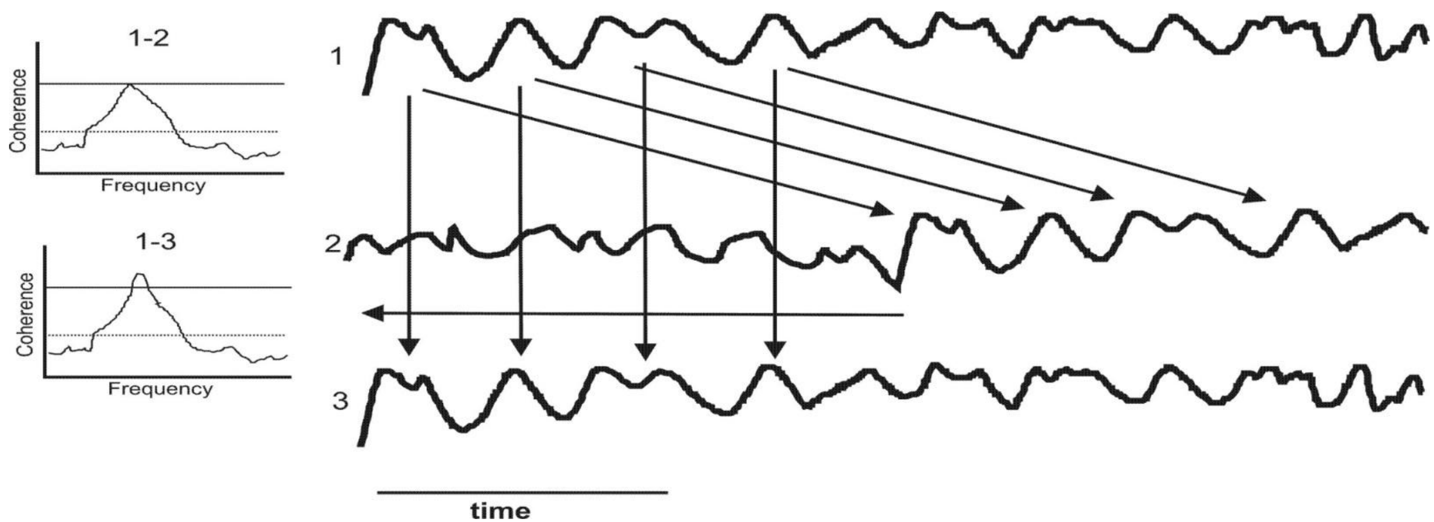
$$\delta = \max_{\delta} obj(\delta) \quad (5.14)$$

Motivation of delay estimation using maximising coherence method

The traditional method of estimating delay ➜ broad-band coherence ➜ enough data points to fit a straight line ➜ If two time series show significant coherence ➜ over a wide frequency band ➜ minimal phase relation ➜ estimate of the time delay from the phase estimate ➜ cannot be used to estimate delay from narrow band coherent signals.

The delay between the two time series is given by

$$\delta = \underbrace{\arg\max}_{\tau} C'(\tau)_{\omega_0} \qquad (19)$$



Maximising coherence method

By shifting one time series $x(t)$ by a time lag τ ➡ the length of the shifted time series $x(t)$ will be less than the unshifted time series $y(t)$ by $N-\tau$ data points ➡ extra data points discarded ➡ coherence is a relative measure with the length ➡ considered only length of the data which is an integer multiple of N ➡ $C(\tau)_{\omega_0}$ same confidence limit ➡ ensures maximum value in coherence due to time delay.

Significance of a time delay τ:

- ➡ Checked by the confidence limit.
- ➡ Variability of the time delay is obtained by the so-called surrogate analysis.
- ➡ error bars for the estimated delay are calculated using this anaylsis.
- ➡ Surrogates are generated using one of the basic assumptions of spectral analysis, namely the disjoint segments of a time series are independent.

Surrogate analysis

- The disjoint data segments D of one of the time series $x(t)$ from which the time-delayed information is assumed to flow to the other time series $y(t)$ is shuffled randomly.
- The cross spectrum is different but the original spectra of both the time series are unchanged.
- 19-100 different surrogates can be estimated for all the analyses and determine the time-delay depending coherence function $C(\tau)_{\omega_0}^{surr}$ for all the realisations.
- Null-Hypothesis ➡ Coherence calculated between the two time series is obtained due to spurious correlations.
- The significance $S(\tau)$ of difference between $C(\tau)_{\omega_0}$ and $C(\tau)_{\omega_0}^{surr}$ is calculated as

$$S(\tau) = \frac{\left| C(\tau)_{\omega_0} - \left\langle C(\tau)_{\omega_0}^{surr} \right\rangle \right|}{\upsilon[C(\tau)_{\omega_0}^{surr}]} \qquad (20)$$

where $\langle \cdot \rangle$ indicates the average over different realisations of surrogates and $\upsilon[.]$ indicates the standard deviation between different realisations.

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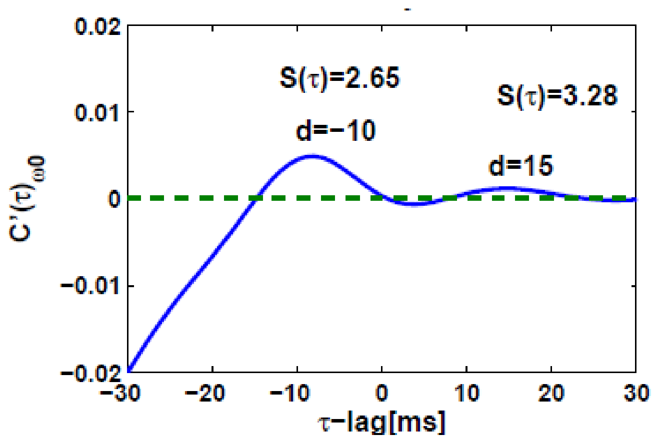
Surrogate analysis

- If $S(\tau) > 2$, the null hypothesis is rejected and $C(\tau)_{\omega_0}$ is used in further analysis.
- Error bars are calculated by subtracting $C(\tau)_{\omega_0}^{surr}$ from $C(\tau)_{\omega_0}$ and determine the delay for every surrogate subtracted realisation.
- In order to have clarity of the delay, we plot $C'(\tau)_{\omega_0}$ in which the maximum value of coherence $C(\omega_0)$ is at $\tau = 0$ and the average of all the surrogates at $\tau = 0$ is given by $\langle C(\omega_0)^{surr} \rangle$.
- Thus $C'(\tau)_{\omega_0}$ will pass through a zero value at $\tau = 0$ and will attain its maximum at $\tau = \delta$;

$$C'(\tau)_{\omega_0} = \left[C(\tau)_{\omega_0} - \langle C(\tau)_{\omega_0}^{surr} \rangle \right] - \left[C(\omega_0) - \langle C(\omega_0)^{surr} \rangle \right] \qquad (21)$$



Topics of Presentation

Topics	Student Name
1) Signal processing is MEG	
2) Mapping the SNR of cortical sources in MEG/EEG	
3) Comparison of EEG and MEG in source level	
4) FEM for forward Modelling	
5) Sparse source imaging	
6) Eigenspace projection beamformers	
7) MEG/EEG source reconstruction using NUTMEG	
8) Mapping human brain with MEG and EEG	
9) Data driven time frequency analysis	
10) Power envelope correlations – source analysis	

Topics of Presentation

Topics	Student Name
11) Overview on artifact correction algorithms – Gradient	
12) Overview on artifact correction analysis – BCG artifact	
13) Spatial-temporal signal separation method	
14) Phase amplitude coupling between neuronal oscillations of different frequencies	
15) Driver Fatigue: EEG and pschological assessment	

Topics of Presentation

Topics	Student Name
16) Review on directionality methods	
17) Review of brain connectivity in EEG/MEG	
18) Resting state FMRI	
19) New and emerging techniques for brain mapping	
20) Analyzing effective connectivity in FMRI	
21) NIRS development and field of application	