Signal Processing for Medical Applications –

Frequency Domain Analyses

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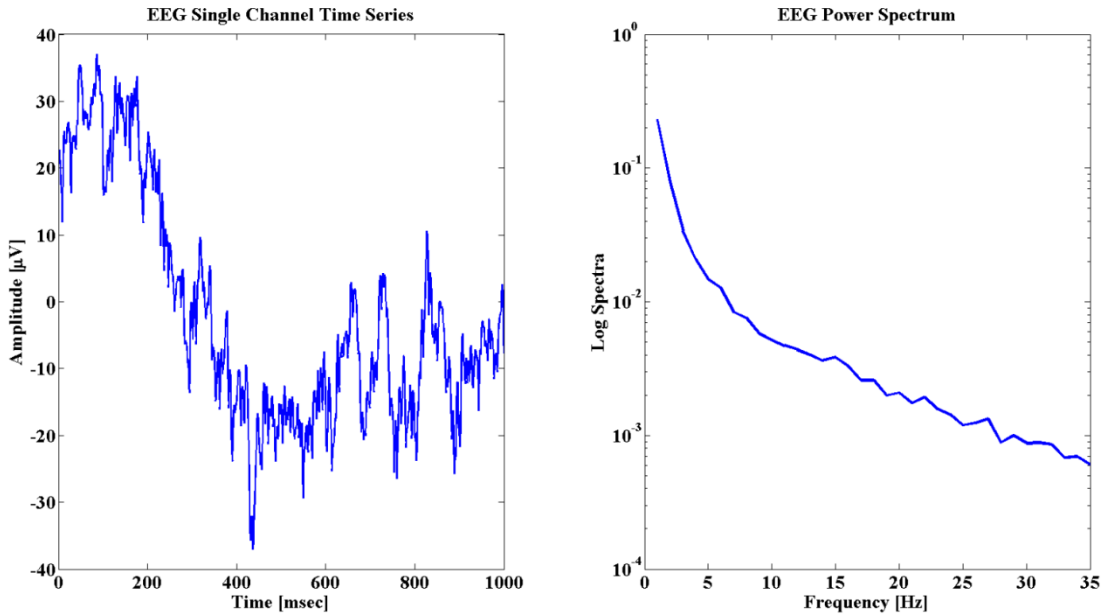
Power spectrum

Let $x(t)$ be a data set of length N which is divided into M disjoint segments of length D so that $N = DM$. The power spectrum $S_{xx}(\omega)$ of the signal $x(t)$ is computed as the Fourier transform of the autocorrelation function in each window.

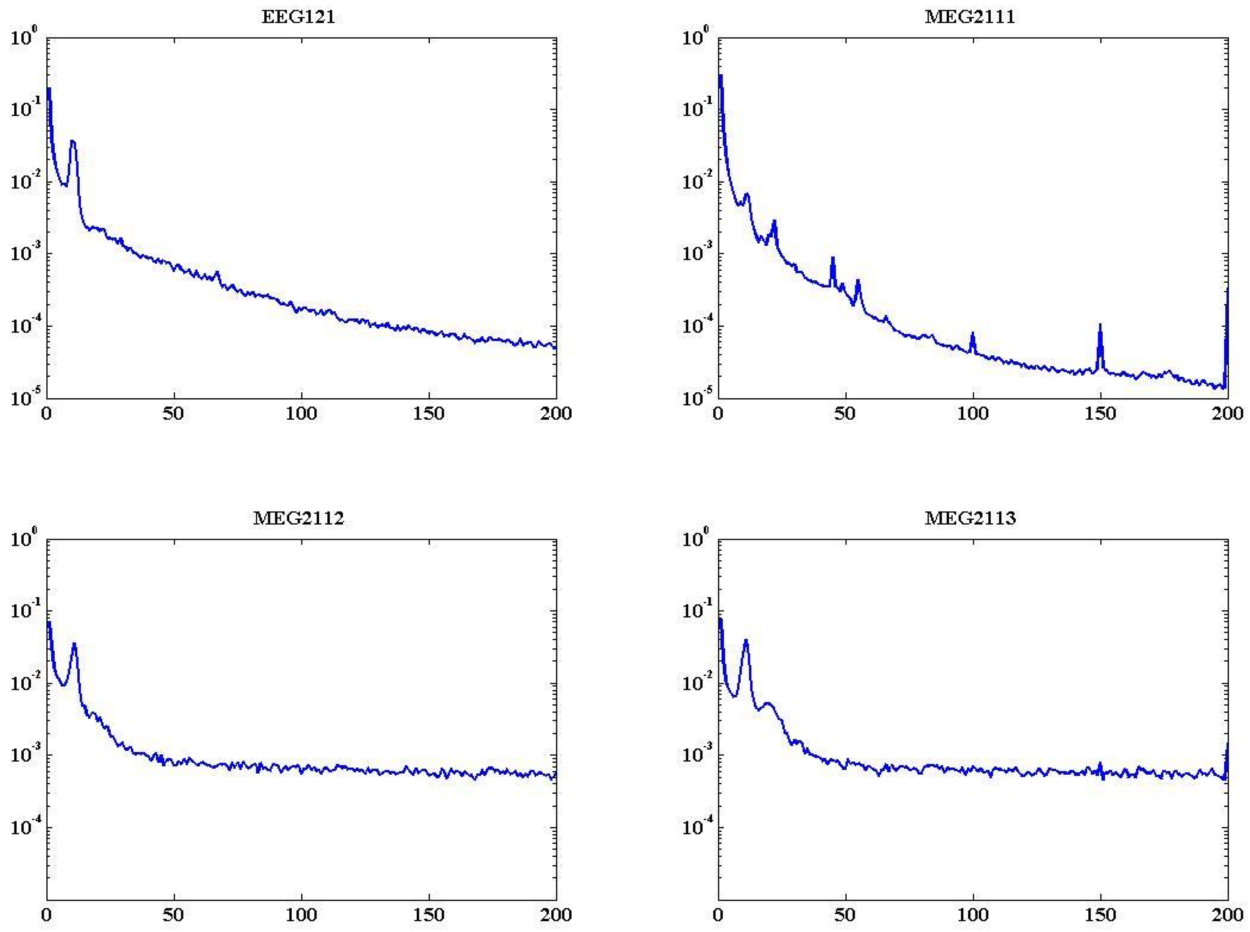
Ex:

If $x(t)$ is a time series of length N = 10,000 data points. f_s = 1 KHz.

$$D = ? \quad M = ?$$



Power spectrum – Eyes Closed – MEG /EEG



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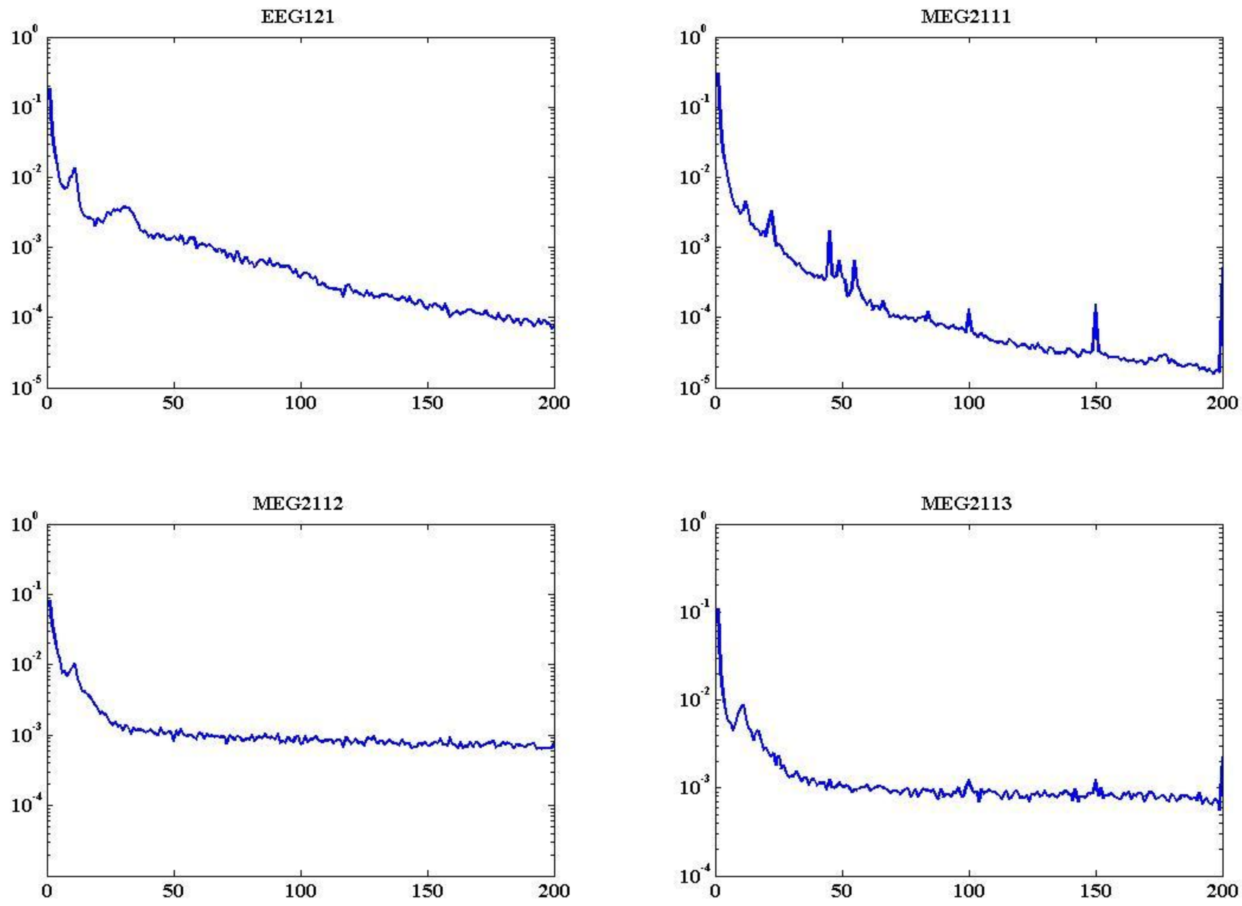
Power spectrum – Eyes Open – MEG /EEG



Power spectrum – MEG /EEG

- Issues in recording and analysing very low frequency EEG/MEG activity

- Infra slow (0.01 to 0.1 Hz) signal recording requires genuine DC-coupled amplifiers with high input impedance, high DC stability and a wide dynamic range.

- DC drift which is superimposed on any meaningful event related slow activity can be an issue unless amplifiers are reset every three minutes to ensure that the signal is kept in the optimal range of the amplifier throughout the recording.

- EEG/MEG activity belongs to a broad class of physical signals which arise from a so-called $1/f$ process. Such signals have a power law relationship of the form:

$$S_{xx}(f) = \frac{1}{|f|^{\gamma}} \qquad (3.1)$$

where $S_{xx}(f)$ is the power spectral density, f is the frequency and γ is the spectral parameter which is usually close to 1 but can lie in the range $0 < \gamma < 2$ and could be greater than 2 in the presence of noise sources.

Power spectrum – MEG /EEG

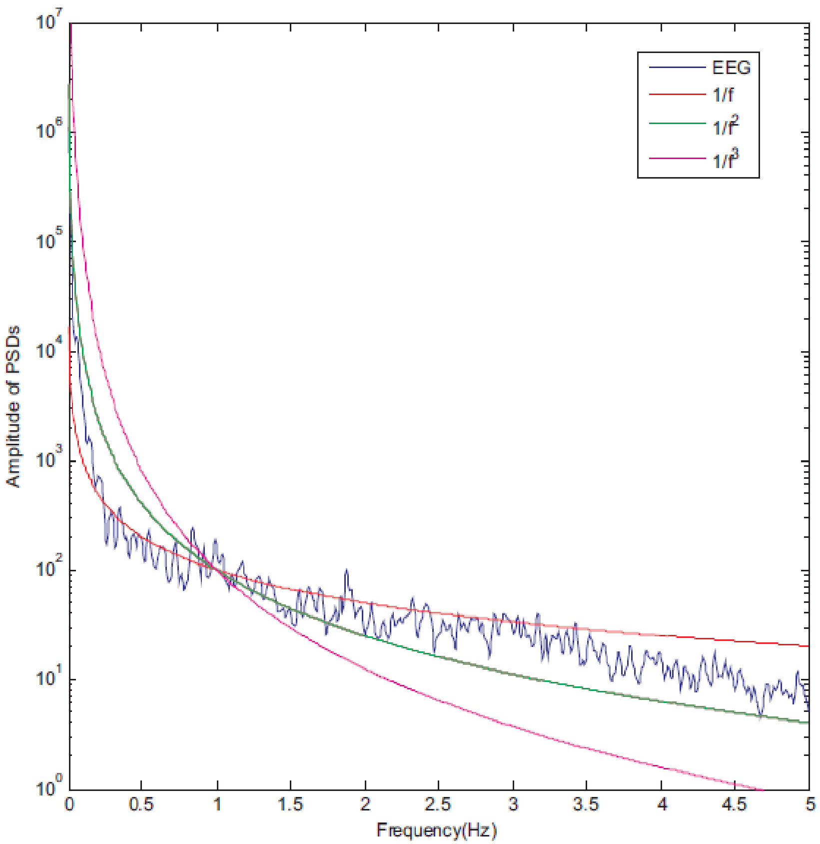
- Basics of EEG/MEG

A large cluster of neurons, each generating a unit activity, forms a functional network which is held together by the neurons synchronisation that ensures activity control.

Neurons in turn attract further neurons and the oscillation amplitude increases.

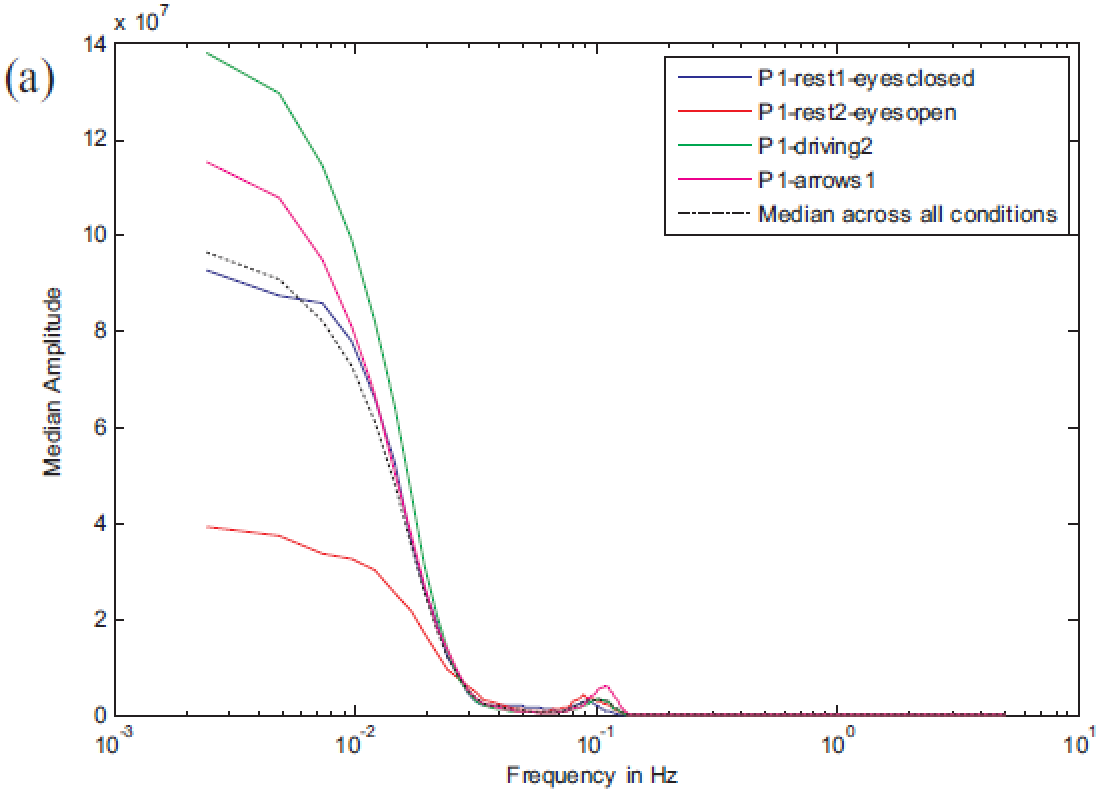
Period of oscillation depends on size of the neuronal cluster that constitutes a given cycle.

Large neuronal areas are associated with slow, high amplitude oscillations whereas a small, localised concentration of neurons gives rise to High frequency, low amplitude signals.



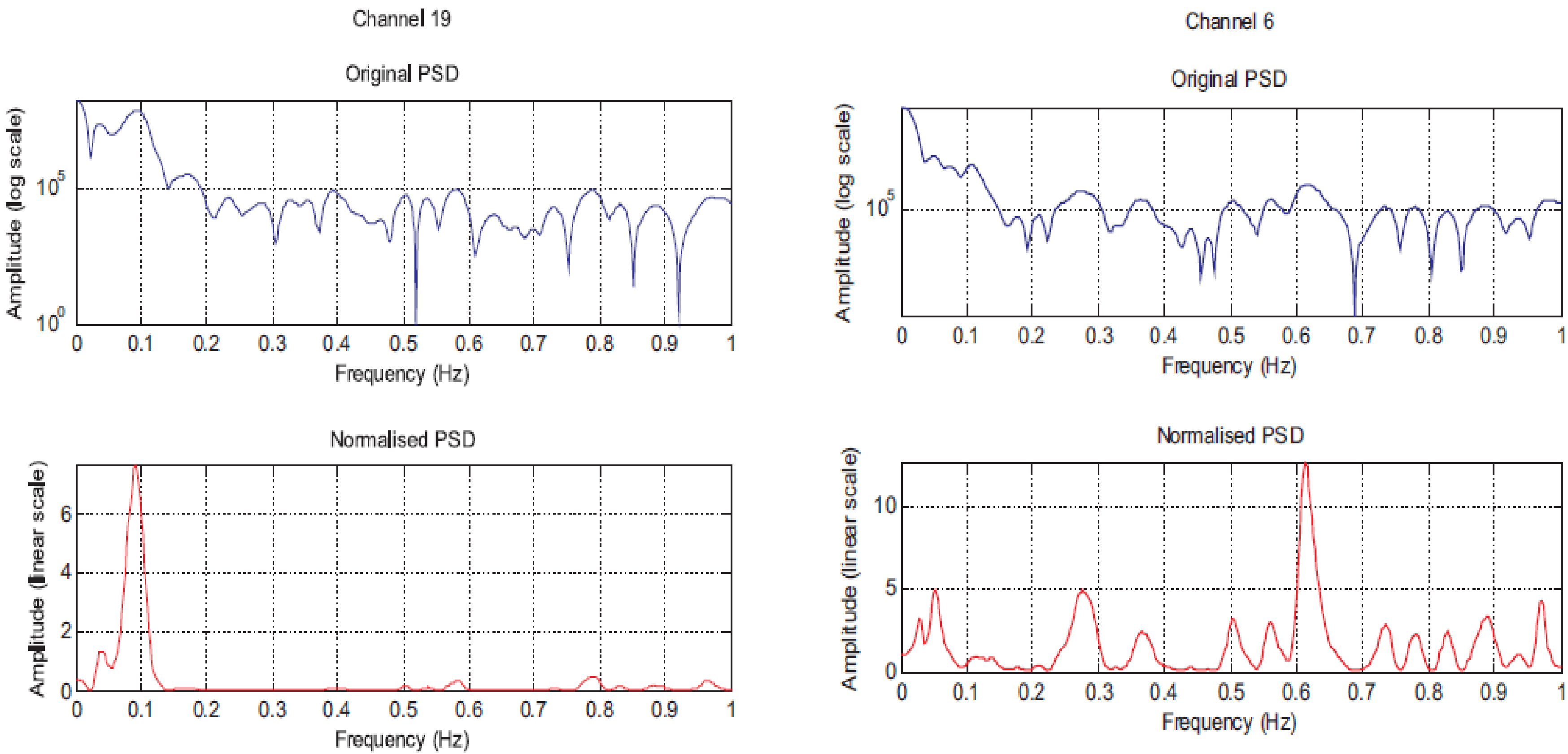
Power spectrum – MEG /EEG

Normalisation in frequency domain



- The median across all time windows was found for every frequency point.
- Thus a graph of the median PSD value for each frequency band was estimated.
- The same procedure will be repeated for all the EEG channels.
- Then the overall median of the median PSD curves across all the conditions which is the normalisation curve.

Power spectrum – MEG /EEG



Power spectrum – MEG /EEG

Normalisation in time domain

- The normalisation can be achieved by passing the signal through a filter that cancels the $1/f^{\gamma}$ spectral behaviour to any spectral analysis.

- This inverse filter can be established by modelling the normalisation curve by an autoregressive(AR) or a moving average (MA) model and then swapping the coefficients to obtain its inverse. Hence,

$$A/f^{\gamma} \times Bf^{\gamma} \approx AB \qquad (3.2)$$

where A/f^{γ} is the EEG spectrum with the intrinsic $1/f^{\gamma}$ characteristics, Bf^{γ} is the inverse filter contribution and AB is the result of their interaction.

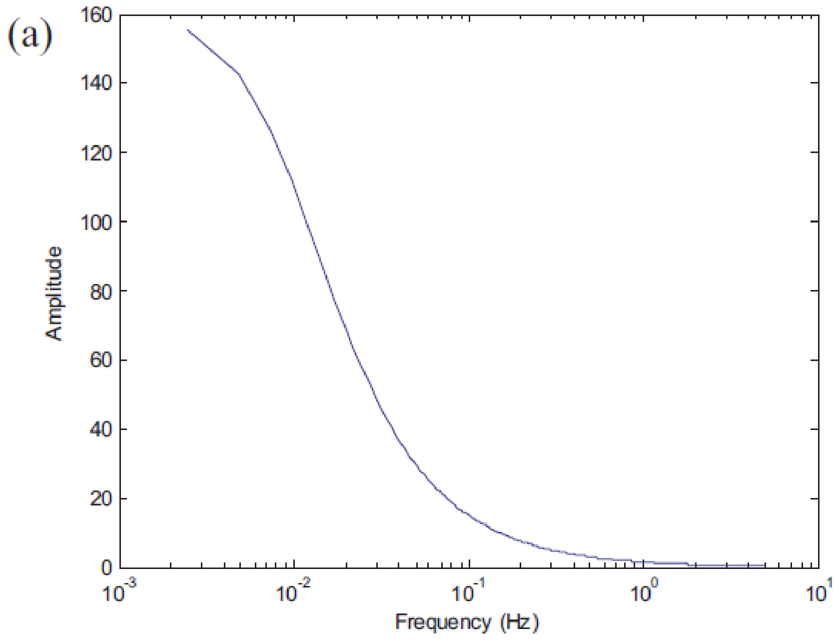
- The $1/f^{\gamma}$ curve can be modelled as a finite impulse response (FIR) model such that the inverse will be an infinite impulse response (IIR) model.

Power spectrum – MEG /EEG

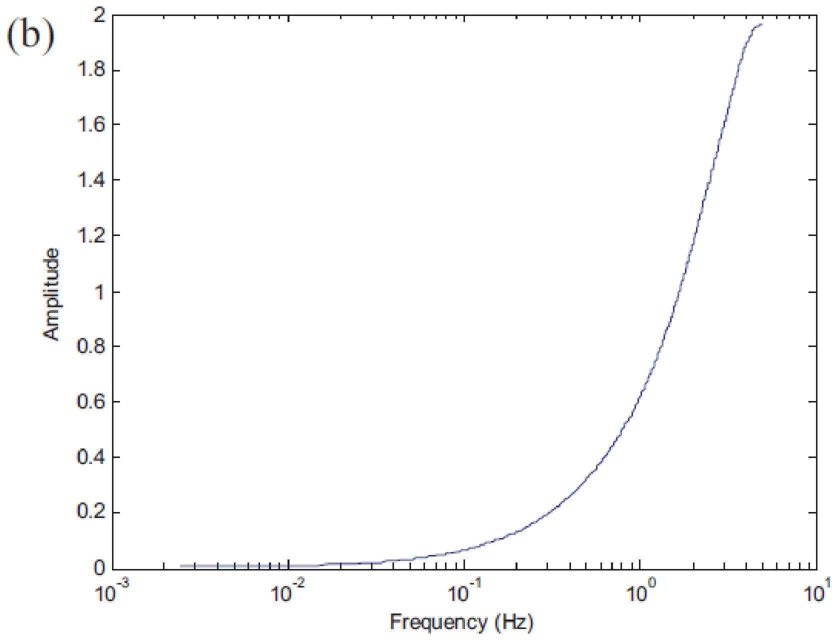
- The problem is the lack of control on the FIR coefficients since these are already predetermined by the shape of the normalisation curve.
- But, if the resultant FIR model is not minimum phase the IIR model stability is a major issue.
- The filter should be a linear phase filter to avoid phase distortion of the input EEG/MEG signal – and a IIR filter will not meet this requirement.
- The other possible approach is that of modelling the normalisation curve as a AR model such that its inverse is an moving average (MA) model and the stability is guaranteed.
- The AR coefficients of an IIR filter are then obtained using the Yule-Walker equations on the absolute value of this time domain signal.

Power spectrum – MEG /EEG

AR model



MA model



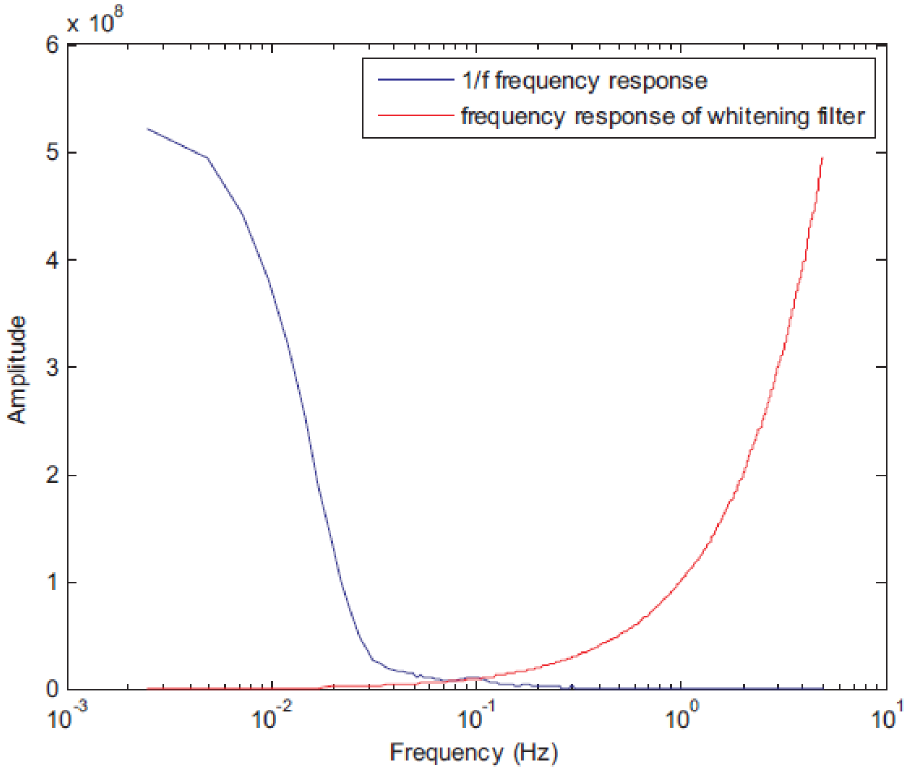
Power spectrum – MEG /EEG

- Although the earlier approach gives the expected results, it is an involved method which depends on the normalisation curve in the frequency domain in order to derive the appropriate inverse filter.

Approximating the inverse filter by a differentiator

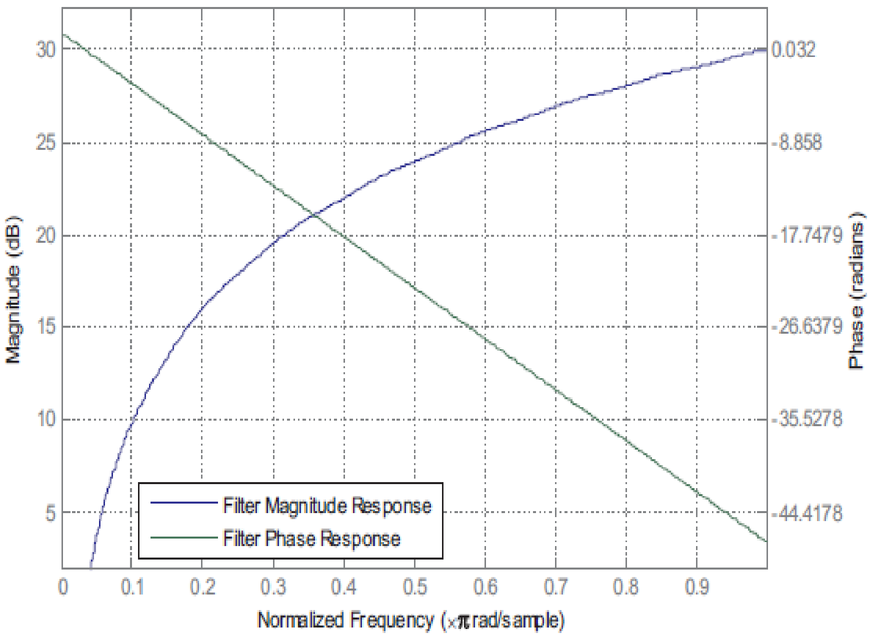
- The normalisation curve can be approximated to be a $1/f$ curve, i.e., setting $\gamma = 1$, and the inverse filter can be obtained by a applying a differentiator.
- The differentiator, with its f frequency response, cancels out the $1/f$ trend of the EEG power spectrum as described by equation (3.2).
- The designed filted should exihibit a linear phase response and a constant group delay.
- The linear phase response makes it easier to compensate for the phase delay at one particular time instant by sample-shifting the pre-reorded data accordingly.

Power spectrum – MEG /EEG

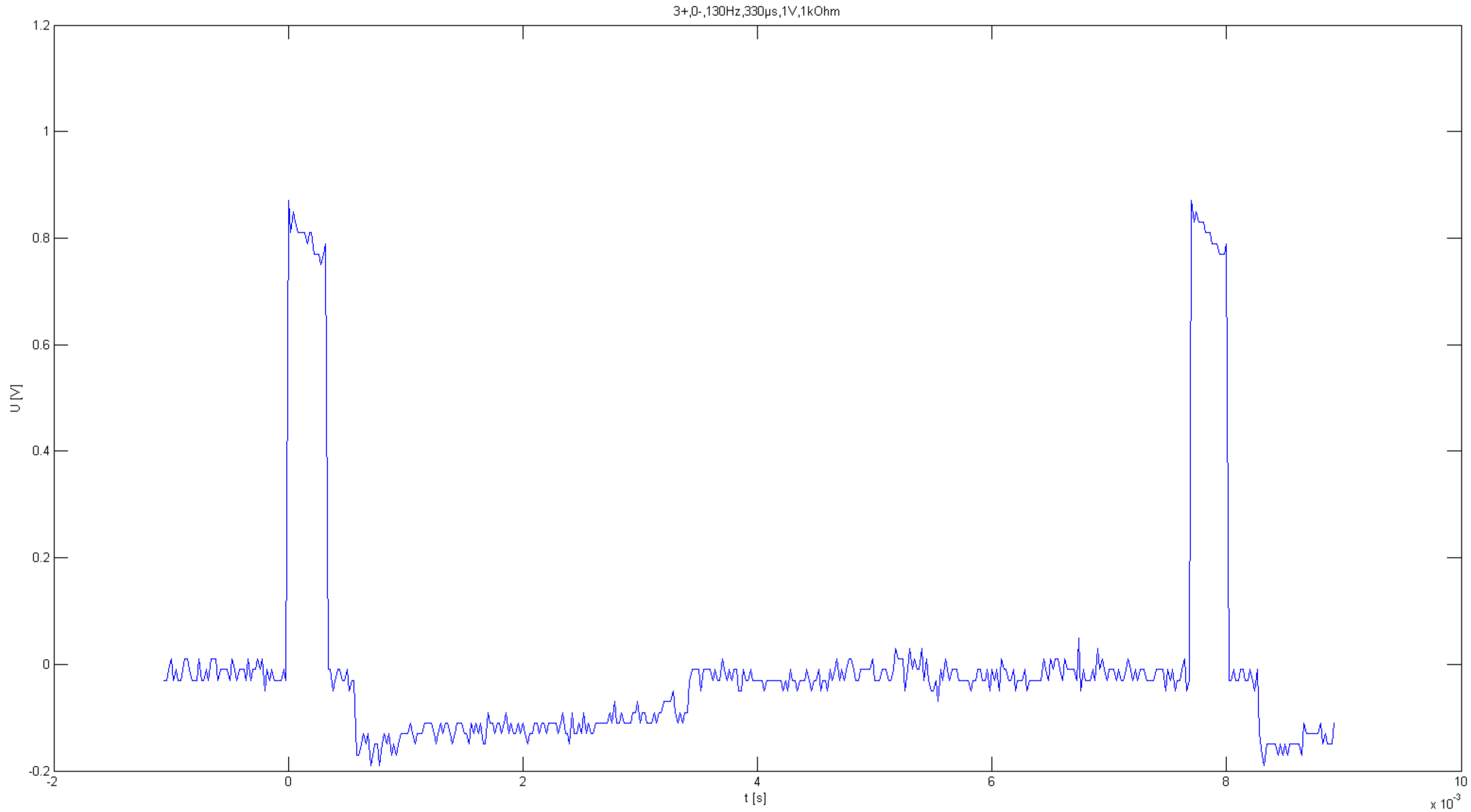


The function of differentiator in spectral normalisation

Magnitude and phase response of the differentiator

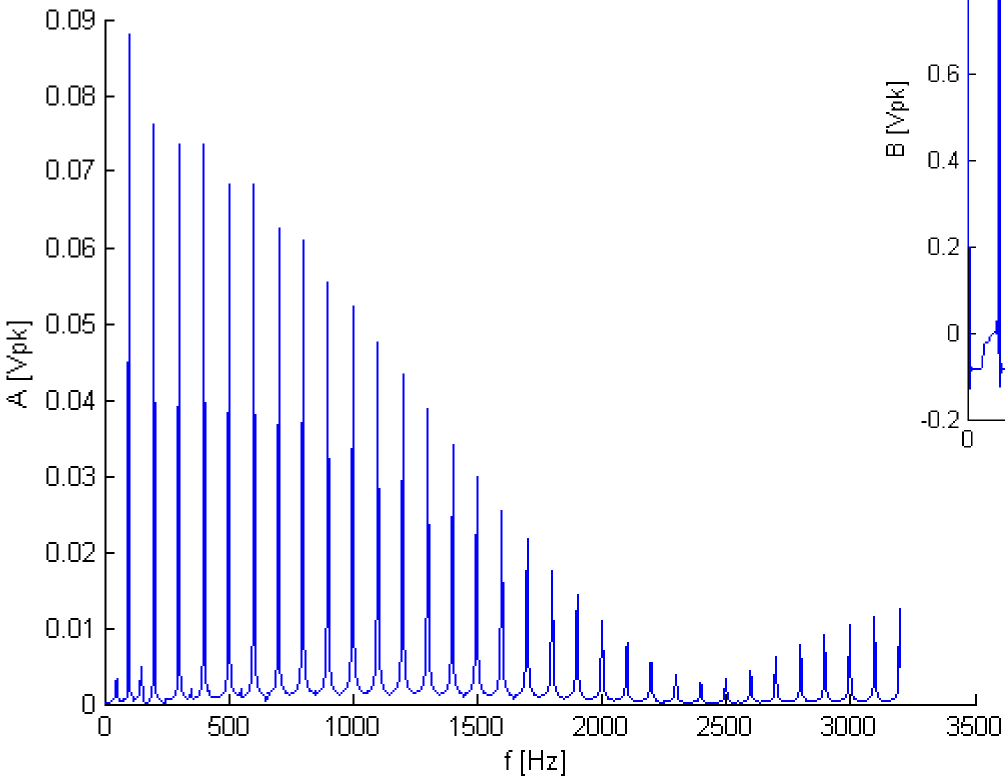


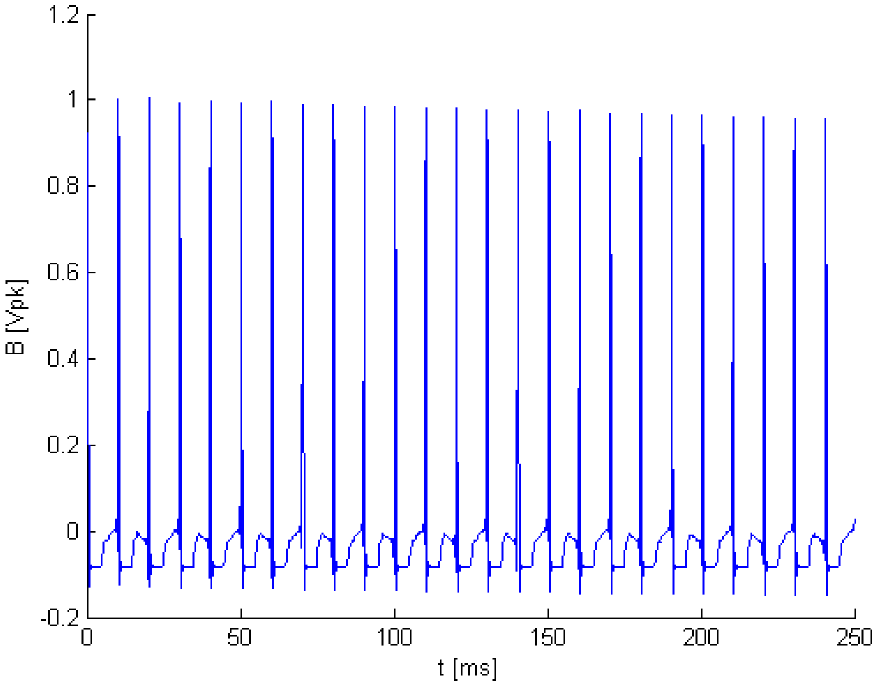
Deep Brain Stimulation



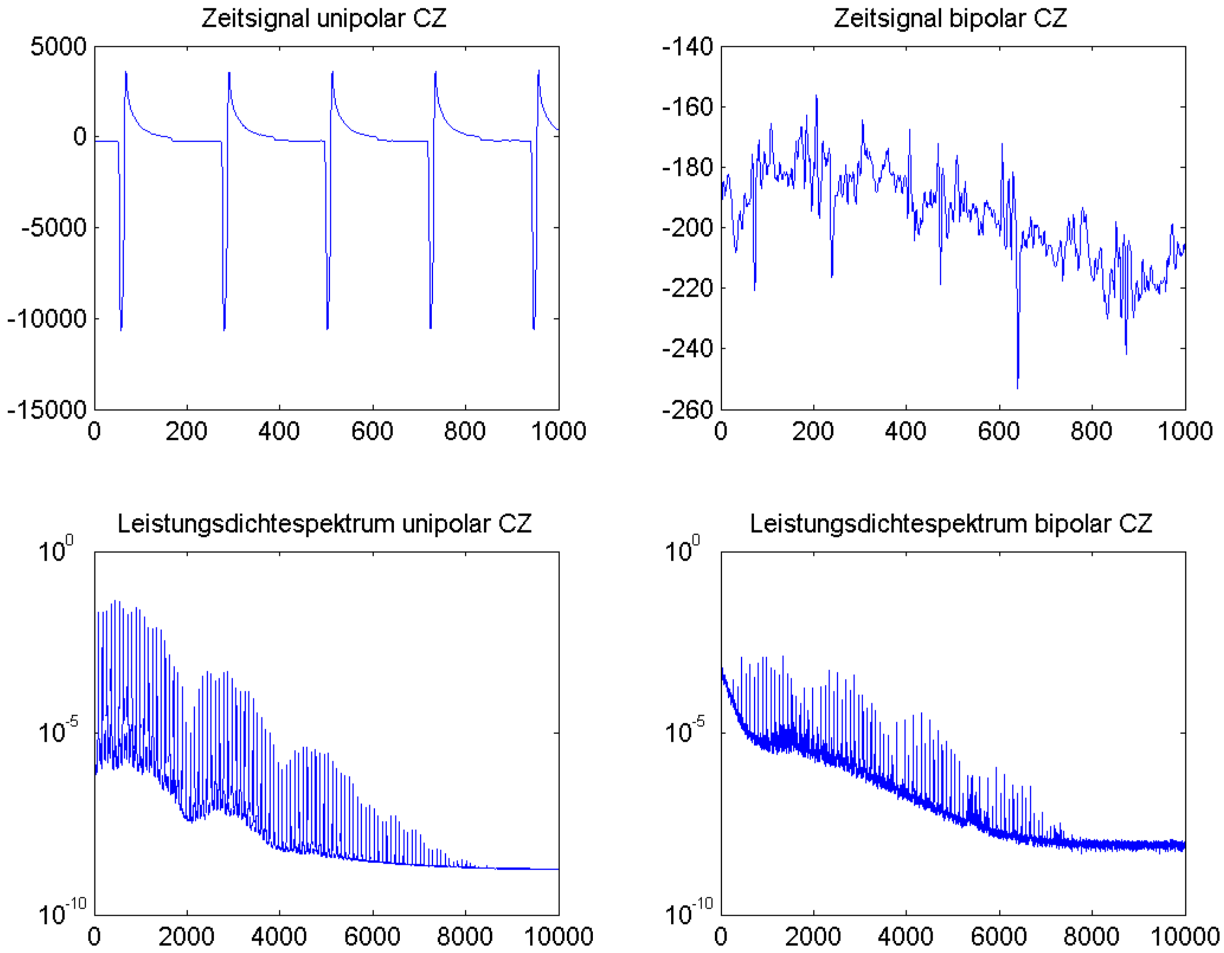
Deep Brain Stimulation

1v -100 Hz- 180 us





Power spectrum – Deep Brain Stimulation - EEG



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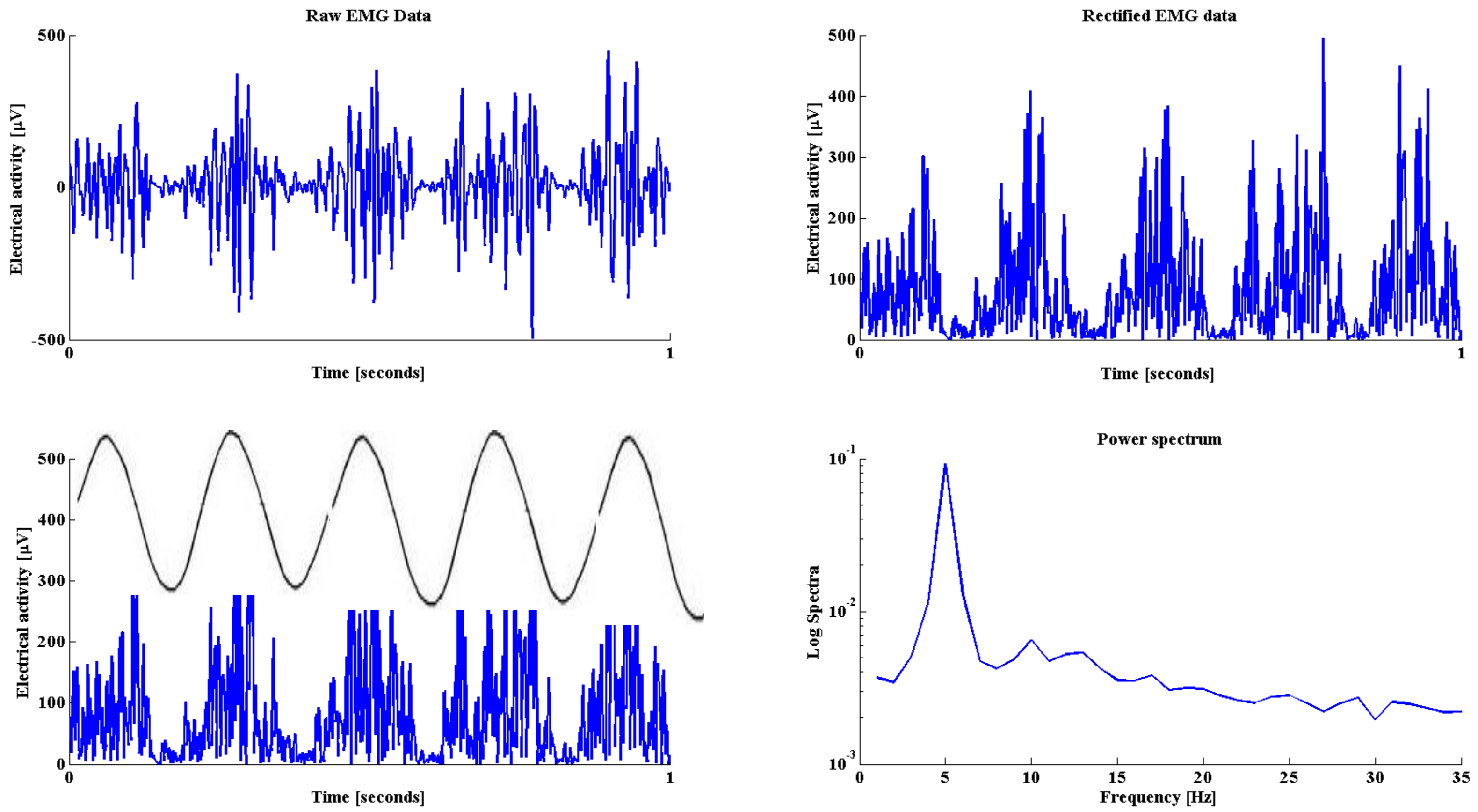
EMG Pre-processing Procedure



Modeling Using Autoregressive Process

Definition

- An autoregressive process is a type of random process which is often used to model different types of biological systems.
- The process should be capable of modelling both the broad-band EEG signal and narrow-band EMG signal with a particular frequency content.
- The AR process is a stochastic process in which the current value of a variable is a linear function of ist own past values, with a white-noise term added.
- The coefficients of the AR process can be used to describe a particular frequency of oscillation.

Modeling Using Autoregressive Process

- In order to find the AR coefficients for a given signal there are various techniques available like the Burg method, modified covariance method, and Yule walker method.

- Here, the Yule–Walker equations will be discussed.

- The AR model of a order G can be written as

$$y[n] = -\sum_{k=1}^{G} a_k \, y[n-k] + \eta(n) \qquad (3.3)$$

- The process is termed an autoregression in that the sequence $y[n]$ is linear regression on itself with $\eta[n]$ representing a Gaussian white-noise term with zero mean and unit variance.

- The Yule- Walker equations are derived using a vector-space view point.

Modeling Using Autoregressive Process

- The inner product is defined as

$$\langle y, x \rangle = \xi(y^* x) \tag{3.4}$$

so that the squared norm of a vector is

$$\|y\|^2 = \langle y, y \rangle = \xi\left(|y|^2\right) = \mathrm{var}(y) \tag{3.5}$$

The linear-prediction problem is to find the optimal set of coefficients $a_1, a_2, \ldots, a_g$ such that

$$\hat{y}[n] = -\sum_{k=1}^{G} a_k y[n-k] \tag{3.6}$$

is the „best“ predictor of $y[n]$ given $y[n-1], y[n-2], \ldots, y[n-g]$. In anticipation of the result that the linear prediction coefficients are equal to the $AR(G)$ parameters, a_k has been used to denote the prediction coefficients.

Modeling Using Autoregressive Process

- „Best“ means that the mean-square error

$$\sigma = \xi\left(\left\| y[n] - \hat{y}[n] \right\|^2\right) = \left\| y[n] - \hat{y}[n] \right\|^2 \tag{3.7}$$

is minimized. By the orthogonality principle, the optimal predictor is found by requiring the error vector $y[n]-\hat{y}[n]$ to be orthogonal to the subspace spanned by $y[n]$ given $y[n-1], y[n-2], \ldots y[n-g]$ or

$$\left\langle y[n-k], y[n] - \hat{y}[n] \right\rangle = 0, k = 1, 2, \ldots, g \tag{3.8}$$

- Inserting equation (3.6) in equation (3.8) and with standard properties of inner products, we obtain

$$\left\langle y[n-k], y[n] + \sum_{k=1}^{G} a_l y[n-l] \right\rangle = 0 \tag{3.9}$$

$$\sum_{l=1}^{G} a_l \left\langle y[n-k], y[n-l] \right\rangle = -\left\langle y[n-k], y[n] \right\rangle \tag{3.10}$$

Modeling Using Autoregressive Process

- Evaluating the inner product as

$$\sum_{l=1}^{G} a_l \varepsilon(y^*[n-k]\, y[n-l] = -\varepsilon(y^*[n-k]\, y[n]) \qquad (3.11)$$

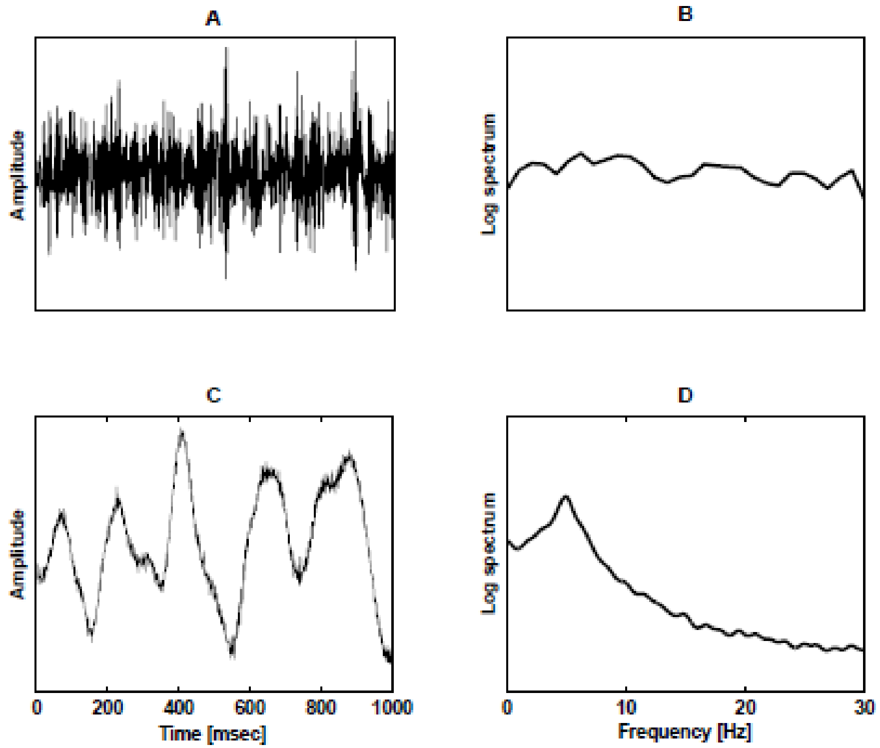
results in

$$\sum_{l=1}^{G} a_l\, r_{yy}[k-l] = -r_{yy}[k] k = 1,2,\ldots p. \qquad (3.12)$$

- This is the set of Yule-walker equations. The solution of (3.12) provides the optimal set of coefficients to predict $y[n]$ as a linear combination of $y[n-1], y[n-2],\ldots y[n-g]$.

Modeling Using Autoregressive Process

- The model system to test the spectral analysis methods are usually designed using the AR of order 2 termed as AR(2). In a physical point of view, an AR(2) process can be interpreted as a stochstically driven, damped, resonant, harmonic oscillator with period and relaxation time determined by the parameters a_1 and a_2.



Modeling Using Autoregressive second order (AR2) Process

- Why AR2 Process:

The way of modeling signals using low order autoregressive model (which delivers a specific rhythm with a noise), and then introducing time shifts of such signal with simultaneous addition of another noise component is frequently used in biomedical signal simulation.

The system is defined by:

$$y(t) = a_1 y(t-1) + a_2 y(t-2) + \eta(t) \qquad (3.13)$$

where a_1 and a_2 is calculated as

$$a_1 = 2 \times \cos(2 \times \pi / T) \times \exp(-1/\tau) \qquad (3.14)$$

$$a_2 = -\exp(-2/\tau) \qquad (3.15)$$

$y(t)$ - output signal; $\eta(t)$ - noise; τ - relaxation time; T - Oscillation period

Modeling Using Autoregressive second order (AR2) Process

For modeling a signal $y(t)$ which resembles a EMG signal and has a peak at 5 Hz in the power spectrum.

If the f_s = 1Khz; a_1 =1.974; a_2 =-0.9802; T = ?; τ = ?;