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Advanced Signals and Systems – Extensions

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Contents of the Lecture

Entire Semester:

- Introduction
- Discrete signals and random processes
- Spectra
- Discrete systems
- Idealized linear, shift-invariant systems
- Hilbert transform
- State-space description and system realizations
- ***Extensions***

Extensions

- All-pass filters

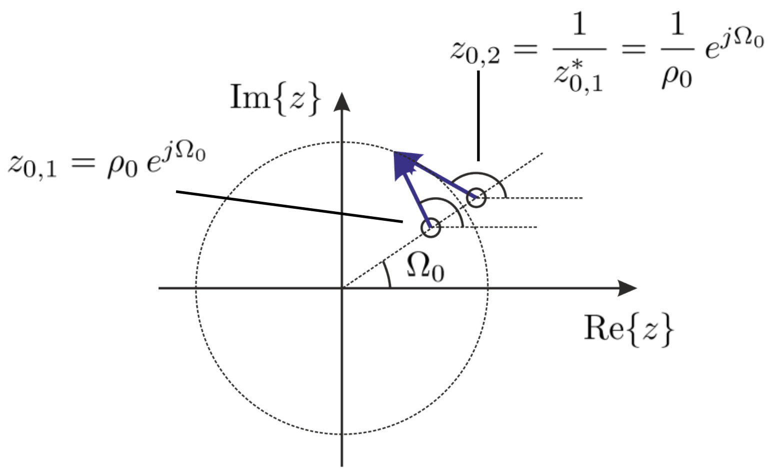
All-Pass Filters – Part 1

Symmetry of poles and/or zeros

If some zeros or poles show ***certain magnitude or phase relations***, special types of systems can be created. We will start first with ***pairs of zeros*** that have the following restriction:

$$z_{0,2} = \frac{1}{z_{0,1}^*}.$$

In a pole-zero plot we obtain the following behavior:



All-Pass Filters – Part 2

Symmetry of poles and/or zeros (continued)

If we look at the ***magnitudes*** of both zero contributions we obatin:

$$|e^{j\Omega} - z_{0,1}| = \rho_0\, |e^{j\Omega} - z_{0,2}| \;\; \forall\, \Omega.$$

… derivation on the blackboard …

For the ***phase*** of the connection of both zeros we get:

$$\begin{aligned}
& \arg\Big\{ \big[e^{j\Omega} - \rho_0\, e^{j\Omega_0}\big]\, \big[e^{j\Omega} - \frac{1}{\rho_0}\, e^{j\Omega_0}\big] \Big\} \\
= \;& \arg\Big\{ e^{j2\Omega} + e^{j2\Omega_0} - (\rho_0 + \frac{1}{\rho_0})\, e^{j(\Omega+\Omega_0)} \Big\} \\
= \;& \arg\Big\{ e^{j(\Omega+\Omega_0)}\, \big[e^{j(\Omega-\Omega_0)} + e^{-j(\Omega-\Omega_0)} - (\rho_0 + \frac{1}{\rho_0})\big] \Big\} \\
= \;& \arg\Big\{ e^{j(\Omega+\Omega_0)}\, \underbrace{\big[2\cos(\Omega-\Omega_0) - (\rho_0 + \frac{1}{\rho_0})\big]}_{\text{real}} \Big\} \\
= \;& \arg\Big\{ e^{j(\Omega+\Omega_0)} \Big\} \\
= \;& \Omega + \Omega_0.
\end{aligned}$$

(affin) linear with respect to the normalized frequency!

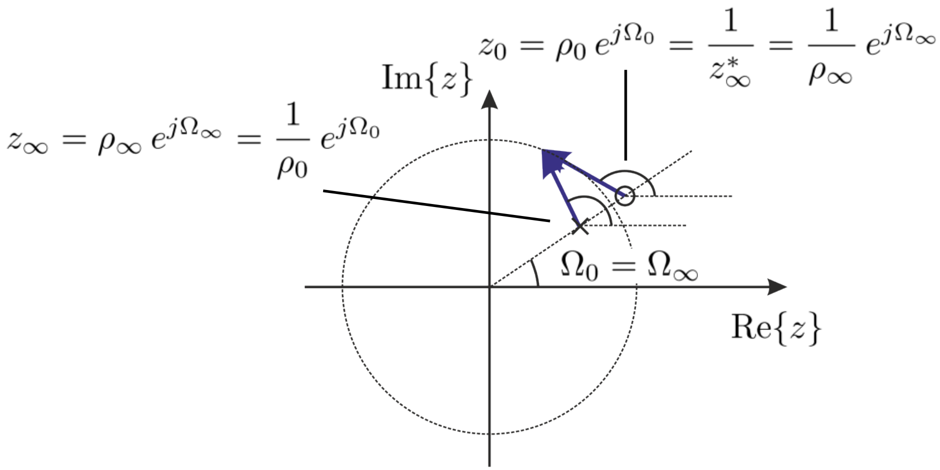
All-Pass Filters – Part 3

Symmetry of poles and/or zeros (continued)

Now we have a look at pole-zero combinations that exhibit the following relation:

$$z_0 = \frac{1}{z_\infty^*}.$$

In pole-zero plots those combinations look like this:



Be aware that stability has to be ensured, meaning that all poles have to be inside the unit circle and thus all zeros must be outside of it!

All-Pass Filters – Part 4

Symmetry of poles and/or zeros (continued)

For the magnitudes we get:

$$\frac{|e^{j\Omega} - z_0|}{|e^{j\Omega} - z_\infty|} = \rho_0 \;\; \forall\, \Omega.$$

Less important are here the phase relations. The magnitude relation that is shown before, however, leads to a ***constant magnitude contribution of the pole-zero combination*** (not dependent on frequency).

Systems with such a pole-zero relation exhibit a constant magnitude frequency response. They are called ***all-pass systems***, since all frequency can pass such a system with the same gain.

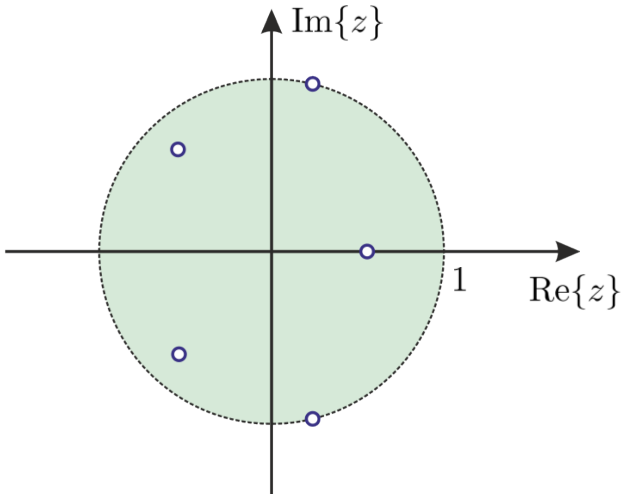
All-Pass Filters – Part 5

Minimum-phase Systems

If a system has all its zeros within the unit circle

$$\left|z_{0,\mu}\right| \leq 1 \;\; \forall \, \mu,$$

it is called a ***minimum-phase system***.



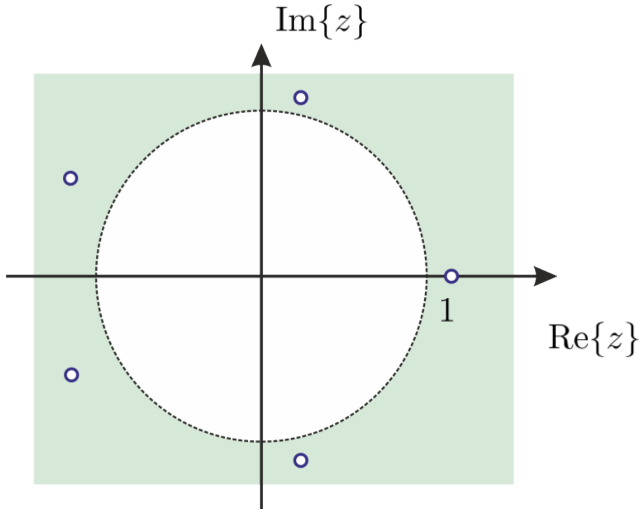
All-Pass Filters – Part 6

Maximum-phase Systems

Systems that have all zeros outside the unit circle

$$\left|z_{0,\mu}\right| > 1 \ \forall \mu,$$

are called ***maximum-phase systems***.



Consequence: Stable all-pass filters are maximum-phase systems.

All-Pass Filters – Part 7

Mixed-phase Systems

Systems with zeros inside and outside the unit circle are called ***mixed-phase systems***. They can be ***decomposed into a minimum-phase system and an all-pass system***.

- For the magnitude frequency response we get:

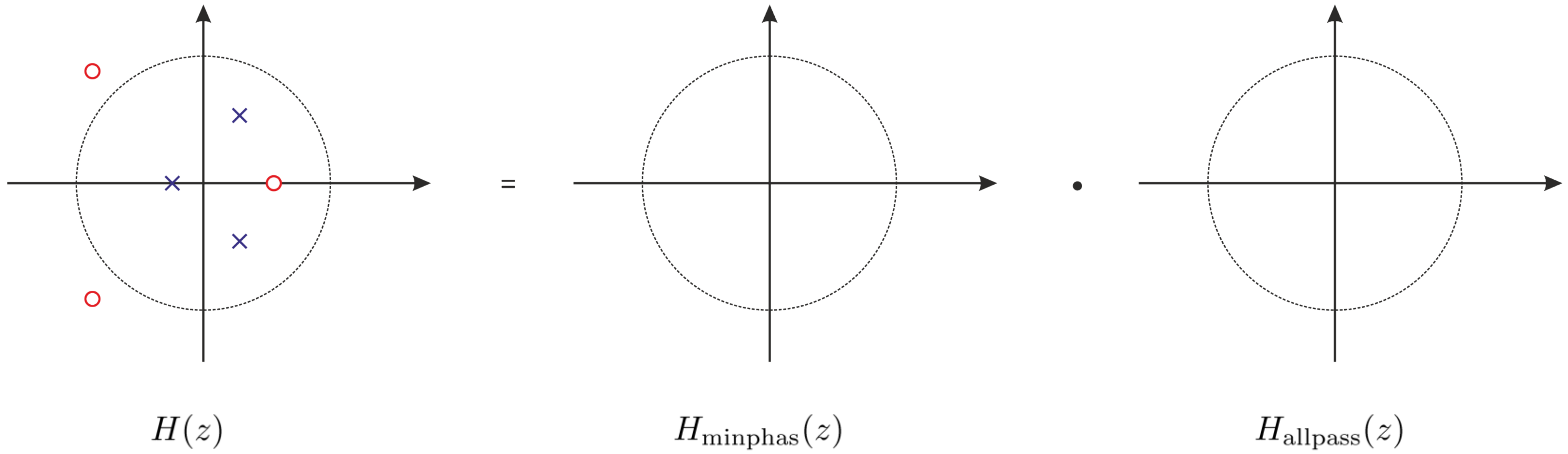
$$\left|H(e^{j\Omega})\right| = \left|H_{\text{minphas}}(e^{j\Omega})\right| \underbrace{\left|H_{\text{allpass}}(e^{j\Omega})\right|}_{=1\,\forall\Omega} = \left|H_{\text{minphas}}(e^{j\Omega})\right|.$$

- The additional poles and zeros in $H_{\text{minphas}}(e^{j\Omega})$ compensate with the corresponding poles and zeros of the all-pass system $H_{\text{allpass}}(e^{j\Omega})$ after cascading both systems.

All-Pass Filters – Part 8

Mixed-phase Systems (continued)

Example of a mixed-phase system:



Please determine the minimum-phase and the all-pass part of the system!

Solution on the blackboard (after individual trials first)

Contents of the Extension Part

This part:

- All-pass filters

No next part – that's it …

Enjoy applying your new knowledge – in the upcoming lectures, during a lab, while working on your thesis and most importantly during your profession as an engineer.

The DSS team