Advanced Signals and Systems – Idealized Linear, Shift-invariant Systems

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System Description – Part 1

**Background:**

Up to now we were focused on the description of deterministic signals and stochastic processes. In addition to that we analyzed the reaction of systems (mostly linear and shift-invariant ones) on signals or processes.

Now we will focus on the following questions:

- Which form do the *characteristic functions of systems* (transfer function, impulse response, etc.) have?
- How does a specific type or form of, e.g., a frequency response influences the impulse response and how does the output signal and its properties change?

We will treat first *individual effects*. Afterward we will investigate differences that appear if we do not have an “ideal system behavior” any more. However, we will not mention yet, if the resulting systems can really be realized.

*The term „ideal behavior“ of a system usually means a distortionless transmission, meaning that the input signals are passed to the output without noticeable difference.*
A distortion-free system usually means the following:

\[ y(n) = v(n) \]

(if no change of the output signal \( y(n) \) is desired). Since real systems do usually need some time to process signals the following demand is more realistic:

\[ y(n) = A v(n - n_0), \]

meaning that at least a delay and a gain is allowed. Without loss of generality we assume for the gain \( A \in \mathbb{R}^+ \) and for the delay \( n_0 \in \mathbb{Z} \) as well as \( n_0 = \text{const} \).
Definitions (continued):

For the *impulse response* of the system we obtain

\[ h_0(n) = A \gamma_0(n - n_0). \]

By summation we obtain the *step response*:

\[ h_{-1}(n) = A \gamma_{-1}(n - n_0). \]

If we chose \( v(n) = e^{j\Omega n} \) as an *input sequence* and we assume a *linear, shift-invariant system* we get for the *output sequence*:

\[ y(n) = A e^{j\Omega(n - n_0)}. \]

As a result we get for the *frequency response*:

\[ H(e^{j\Omega}) = A e^{-j\Omega n_0}. \]
Definitions (continued):

Looking in more detail at the frequency response, we see that we have on the one hand side a \textit{constant magnitude response},

\[ |H(e^{j\Omega})| = A \forall \Omega, \]

and on the other hand a \textit{linear phase response}

\[ \arg\{H(e^{j\Omega})\} = -\Omega n_0. \]

This kind of transmission system is called a \textit{linear-phase all-pass system}. In the same way we obtain in the z-domain

\[ H(z) = A z^{-n_0}. \]

These ideal transmission systems correspond (neglecting the constant gain) to the delay operator that we treated in the previous parts of this lecture!
Idealized Linear, Shift-invariant Systems

Ideal Transmission Systems – Part 4

**Definitions (continued):**

Possible differences from the ideal behavior mentioned before can be classified by the following categories:

- **Magnitude or attenuation distortions:**
  
  \[ |H(e^{j\Omega})| \neq \text{const.} \]

- **Phase- or delay distortions:**
  
  \[ \text{arg}\{H(e^{j\Omega})\} \neq \text{linear.} \]

- **Generic linear distortions:**
  
  \[ |H(e^{j\Omega})| \neq \text{const.} \land \text{arg}\{H(e^{j\Omega})\} \neq \text{linear.} \]

The latter mentioned linear distortion differ – of course – from **non-linear distortions**. They appear, e.g., in systems that are described by non-linear difference equations such as

\[ y(n) = v(n) + v^2(n) + \ldots \]
Definitions (continued):

Remark on phase distortions:

- **Definition of group delay**

  \[ \tau_{\text{group}}(\Omega) = -\frac{d}{d\Omega} \arg\{H(e^{j\Omega})\}. \]

- The **phase- or delay distortions** can be also expressed in terms of ...

  \[ \tau_{\text{group}}(\Omega) \neq \text{const.} \]
In the following we assume to have a linear phase filter. This means that we have

\[ H_{\text{LP, id}}(e^{j\Omega}) = |H_{\text{TP, id}}(e^{j\Omega})| e^{-j\Omega n_0}, \]

For the magnitude response we would like to have:

\[ |H_{\text{LP, id}}(e^{j\Omega})| = \begin{cases} A, & \text{if } |\Omega - \lambda 2\pi| \in [0, \Omega_g], \\ 0, & \text{else}. \end{cases} \]

If such a system is excited with an impulse the output signal will not be an impulse as well (this would only be true if the system would be distortion-free). Instead we obtain the impulse response of an ideal low-pass filter:

\[ h_{0,\text{LP, id}}(n) = A \frac{\Omega_g}{\pi} \frac{\sin(\Omega_g(n - n_0))}{\Omega_g(n - n_0)}. \]
Idealized Linear, Shift-invariant Systems

Attenuation Distortions – Part 2

**Ideal band limitation and ideal low-pass filter (continued):**

In dependence of the cut-off frequency $\Omega_g$, the finite impulse is **widened** to an impulse response with a certain width (e.g. described by $2n_1 = 2\frac{\pi}{\Omega_g}$).

After a convolution with such a system each signal $v(n)$ is **“smeared”** (also called **“leakage”**).

For $\Omega_g \to \pi$ we get $n_1 \to 1$ and $A\frac{\Omega_g}{\pi} \to A$, meaning that the impulse response converges against a weighted impulse sequence,

$$h_{0,LP,\text{id}}(n) \longrightarrow A\gamma_0(n - n_0),$$

meaning that **the low-pass filter becomes a linear-phase all-pass filter (a delay element)**.
Idealized Linear, Shift-invariant Systems

Attenuation Distortions – Part 3

**Ideal band limitation and ideal low-pass filter (continued):**

If an ideal low-pass filter is excited with a unit step sequence $\gamma_{-1}(n)$ the **steep increase of the step sequence around** $n = 0$ **is delayed and “smeared”**. As a consequence of that smearing the steepness is reduced. In addition to that **pre- and post-pulse oscillations** appear (see picture on the right).

For the **step response** we obtain:

$$h_{-1,\text{LP, id}}(n) = \sum_{\kappa = -\infty}^{n} h_{0,\text{LP, id}}(\kappa)$$

$$= A \frac{\Omega_g}{\pi} \sum_{\kappa = -\infty}^{n} \frac{\sin (\Omega_g (\kappa - n_0))}{\Omega_g (\kappa - n_0)}.$$
Ideal band limitation and ideal low-pass filter (continued):

For the properties of an ideal low-pass filter we can summarize:

- According to our start-up assumptions an ideal low-pass filter is **linear** and **shift-invariant**.

- The impulse response $h_{0,LP,id}(n)$ is **infinite**. As a consequence $y(n_0)$ dependent on $v(n)$ with $n \neq n_0$. Thus, we have a **dynamic** system.

- The impulse response $h_{0,LP,id}(n)$ starts having values different from zero before $n = 0$. Thus, we have a **non-causal** and **non-passive** system.

- The sum

$$\sum_{n=-\infty}^{\infty} |h_{0,LP,id}(n)|$$

does not exist in general (but for special cases). As a consequence ideal low-pass filters are **non-stable**.

*Even while violating the „summation condition“ the Fourier transforms of ideal low-pass filters exist. This is because the summation conditions are sufficient but not essential!*
Ideal band limitation and ideal low-pass filter (continued):

Let us investigate now the reaction of the ideal low-pass filter to a white excitation sequence, which can be described by its auto correlation:

\[ s_{vv}(\kappa) = \sigma_v^2 \gamma_0(\kappa). \]

The output auto correlation can be obtained by "double convolution" with the impulse response of the filter and with its mirrored and conjugate complex counterpart:

\[ s_{yy}(\kappa) = s_{vv}(\kappa) * h_{0,LP,id}(\kappa) * h_{0,LP,id}^*(-\kappa) = \sigma_v^2 h_{0,LP,id}(\kappa) * h_{0,LP,id}^*(-\kappa). \]

In the Fourier domain this corresponds to

\[ S_{yy}(e^{j\Omega}) = S_{vv}(e^{j\Omega}) H_{LP,id}(e^{j\Omega}) H_{LP,id}^*(e^{j\Omega}) = \sigma_v^2 |H_{LP,id}(e^{j\Omega})|^2. \]
Ideal band limitation and ideal low-pass filter (continued):

The term $|H_{\text{LP, id}}(e^{j\Omega})|^2$ describes again a single low-pass frequency response (with zero-phase, cut-off frequency $\Omega_g$, and bass-band gain $A^2$). Thus, we obtain for the magnitude squared frequency response:

$$|H_{\text{LP, id}}(e^{j\Omega})|^2 = \begin{cases} A^2, & \text{if } |\Omega - \lambda 2\pi| \in [0, \Omega_g], \\ 0, & \text{else}. \end{cases}$$

Transforming this term by means of an inverse Fourier transform leads to:

$$\mathcal{F}^{-1}\left\{|H_{\text{LP, id}}(e^{j\Omega})|^2\right\} = h_{0, \text{LP, id}}(n) \ast h_{0, \text{LP, id}}^*(-n) = A^2 \frac{\Omega_g}{\pi} \frac{\sin(\Omega_g n)}{\Omega_g n}.$$ 

As a result we obtain for the auto correlation of the output sequence:

$$s_{yy}(\kappa) = \sigma_v^2 A^2 \frac{\Omega_g}{\pi} \frac{\sin(\Omega_g \kappa)}{\Omega_g \kappa}.$$
**Ideal band limitation and ideal low-pass filter (continued):**

For the *auto power spectral density of the output* we obtain

\[
S_{yy}(e^{j\Omega}) = \begin{cases} 
\sigma_v^2 A^2, & \text{if } |\Omega - \lambda 2\pi| \in [0, \Omega_g], \\
0, & \text{else.}
\end{cases}
\]

In the same way we can compute the *cross correlation sequence* and its spectral counterpart, the *cross power spectral density*. We get:

\[
S_{vy}(e^{j\Omega}) = \begin{cases} 
\sigma_v^2 A e^{j\Omega n_0}, & \text{if } |\Omega - \lambda 2\pi| \in [0, \Omega_g], \\
0, & \text{else.}
\end{cases}
\]

\[
s_{vy}(\kappa) = \sigma_v^2 A \frac{\Omega_g}{\pi} \frac{\sin (\Omega_g (\kappa - n_0))}{\Omega_g (\kappa - n_0)}.\]
Some Questions

*Ideal band limitation and ideal low-pass filter (continued):*

Partner work – Please think about the following questions and try to find answers (first group discussions, afterwards broad discussion in the whole group).

- Can you think of applications where an ideal low-pass filter will be part of a system specification (in terms of a system that should be approximated as good as possible)?
  
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- If you take an ideal low-pass filter and move the cut-off frequency towards $\pi$ – what do you get?
  
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Band limitation plus linear pre- and de-emphasis:

Now we will investigate a low-pass filter that differs slightly from the ideal version. The filter has the following magnitude frequency response:

\[
|H_{LP}(e^{j\Omega})| = \begin{cases} 
    A \left(1 + \alpha \frac{\Omega}{\Omega_g}\right), & \text{for } |\Omega + \lambda 2\pi| \leq \Omega_g, \\
    0, & \text{else.}
\end{cases}
\]

Again we will assume a linear-phase filter. Thus, the entire frequency response is defined as:

\[
H_{LP}(e^{j\Omega}) = |H_{LP}(e^{j\Omega})| e^{-j\Omega n_0}.
\]

The background of the following consideration is a better understanding of the property „bandwidth“ in relation with the steepness of the step response.
Band limitation plus linear pre- and de-emphasis (continued):

Computing the impulse response

\[ h_{0,\text{LP}}(n) = \mathcal{F}^{-1}\{H_{\text{LP}}(e^{j\Omega})\} \]

will only be sketched here. In order to avoid complicated integrals we split the frequency response in a rectangle and a triangle first. Second also the triangle is decomposed into two rectangles (see below).

The triangle can be generated by a convolution of two rectangles (both of half the width of the triangle). This corresponds in the time-domain to a multiplication of the corresponding impulse responses.
Band limitation plus linear pre- and de-emphasis (continued):

We obtain for the impulse response:

\[ h_{0,\text{LP}}(n) = (1 + \alpha) A \frac{\Omega_g}{\pi} \sin \left( \frac{\Omega_g (n - n_0)}{\Omega_g (n - n_0)} \right) - \alpha A \frac{\Omega_g}{2\pi} \left[ \sin \left( \frac{\Omega_g}{2} (n - n_0) \right) \right]^2. \]

By summation we obtain the step response:

\[ h_{-1,\text{LP}}(n) = \sum_{\kappa=-\infty}^{n} h_{0,\text{LP}}(\kappa). \]
Idealized Linear, Shift-invariant Systems

Attenuation Distortions – Part 11

**Band limitation plus linear pre- and de-emphasis (continued):**

Sketch of two step responses with different values of $\alpha$:

- **High frequencies are boosted!**
- $\alpha = 0.5$

- **High frequencies are attenuated!**
- $\alpha = -0.5$
Attenuation Distortions – Part 12

**Band limitation plus linear pre- and de-emphasis (continued):**

The degree of temporal „smearing“ can be described by the „width“ (duration) of the main „increase“ part of the step response. This width can be visualized by the gradient of a line fitted to the step response at the point with maximum difference. We get

\[
\frac{A}{K} = \max_n \left\{ h_1(n) - h_1(n - 1) \right\}
\]
\[
= \max_n \left\{ h_0(n) \right\}
\]
\[
= h_0(n_0).
\]

We obtain for the **gradient** of this line (also called rise time):

\[
K = \frac{A}{h_0(n_0)}.
\]

For better understanding a continuous step response has been used in the picture!
Band limitation plus linear pre- and de-emphasis (continued):

If we go back one step and look again on the ideal lowpass filter of the last part, we can define a so-called normalized bandwidth on the one hand side and a rise time on the other hand:

- **Normalized bandwidth**:\
  
  \[ B_{\text{norm}} = \frac{2\Omega_g}{2\pi} = \frac{\Omega_g}{\pi}. \]

- **Rise time**:\
  
  \[ K = \frac{A}{h_0(n_0)} = \frac{\frac{A}{\Omega_g}}{\frac{\pi}{\Omega_g}} = \frac{\pi}{\Omega_g}. \]

We can conclude, that (at least for ideal lowpass filters) the product of rise time and normalized bandwidth is constant:

\[ K \cdot B_{\text{norm}} = 1. \]
**Band limitation plus linear pre- and de-emphasis (continued):**

Now we will do the same investigation for the modified low-pass filters. In order to do so, we will introduce first the so-called *equivalent bandwidth*. For that purpose we design an ideal low-pass filter that exhibits the same area in the magnitude frequency response:

We obtain for the new *cut-off frequency*:

\[
\left(A + A \alpha \frac{1}{2}\right) 2\Omega_g = A 2\Omega'_g \\
\Omega'_g = \Omega_g \left(1 + \frac{\alpha}{2}\right).
\]
Band limitation plus linear pre- and de-emphasis (continued):

Again, we can define now a rise time and a normalized (equivalent) bandwidth for the modified low-pass filter. We get for the ...

- ... normalized, equivalent bandwidth:
  
  \[ B_{\text{norm}} = \frac{\Omega_g (1 + \alpha/2)}{\pi}. \]

- ... rise time:

  \[ K = \frac{\pi}{\Omega_g (1 + \alpha/2)}. \]

As in the last example we get for the product of rise time and normalized, equivalent bandwidth a constant result:

\[ K B_{\text{norm}} = 1. \]
**Band limitation plus linear pre- and de-emphasis (continued):**

Remarks:

- A system with a „quick“ or „steep“ reaction requires a **large bandwidth**. This can be achieved in a straightforward way (using a large $\Omega_s$) or by lifting the frequency response at high frequencies.

- However, as the investigations before show, amplifying larger frequencies leads to increased oscillations and to **overshooting** of the impulse and step responses (this is undesired in several applications). A countermeasure against such overshooting is to attenuate higher frequencies – which „slows down“ the system. Very often a **compromise is made by increasing the bandwidth and attenuating high frequencies at the same time.**
Idealized attenuation ripples:

In the following slides we will again assume a *linear phase response* (a constant group delay). For some applications (such as HiFi amplifiers) it is desirable if the *magnitude response is constant* along the frequency axis. However, in *reality this is hard to achieve*. Very often small derivations from this optimal behavior can be observed. We will model such *attenuation ripples* as sine-shaped fluctuations:

Such a *frequency response* can be described as:

\[
H(e^{j\Omega}) = A \left[ 1 + \alpha \cos \left( 2\pi \frac{\Omega}{\Omega_0} \right) \right] e^{-j\Omega_0 n_0}.
\]
Idealized attenuation ripples (continued):

For computing the corresponding *impulse response* we temporarily neglect the linear phase term and we obtain for the inverse Fourier transform of the zero-phase frequency response:

\[
\tilde{h}_0(n) = A \mathcal{F}^{-1}\left\{1 + \alpha \cos\left(\frac{2\pi}{\Omega_0}\right)\right\}
\]

* ... inserting \( \cos(x) = \frac{1}{2}(e^{jx} + e^{-jx}) \) ...

\[
= A \mathcal{F}^{-1}\left\{1 + \frac{\alpha}{2} e^{j2\pi \frac{\Omega}{\Omega_0}} + \frac{\alpha}{2} e^{-j2\pi \frac{\Omega}{\Omega_0}}\right\}
\]

* ... inverse Fourier transform of harmonic exponentials ...

\[
= A \gamma_0(n) + A\frac{\alpha}{2} \left[\gamma_0(n + \frac{2\pi}{\Omega_0}) + \gamma_0(n - \frac{2\pi}{\Omega_0})\right].
\]

Adding a linear phase term (leading to a shift in time) results finally in the overall *impulse response*:

\[
h_0(n) = A \gamma_0(n - n_0) + A\frac{\alpha}{2} \left[\gamma_0(n - n_0 + \frac{2\pi}{\Omega_0}) + \gamma_0(n - n_0 - \frac{2\pi}{\Omega_0})\right].
\]
**Idealized attenuation ripples (continued):**

Sketch of the *impulse response*:

\[ h_0(n) = A \gamma_0(n - n_0) + A \frac{\alpha}{2} \left( \gamma_0(n - n_0 + \frac{2\pi}{\Omega_0}) + \gamma_0(n - n_0 - \frac{2\pi}{\Omega_0}) \right). \]

The impulse response consists of a *main impulse* at \( n = n_0 \) and two *side* or *echo* impulses appearing \( 2\pi/\Omega_0 \) samples *before and after the main impulse*. The *frequency of the magnitude oscillations* determines the *temporal distance* between the echo impulses. The *height* of these impulses is determined by the *maximum deviation* from the desired value of the magnitude response.
Idealized Linear, Shift-invariant Systems

Attenuation Distortions – Part 20

**Idealized attenuation ripples (continued):**

The corresponding *step response* is obtained by summation of the individual parts of the impulse response. We get:

$$h_{-1}(n) = A\gamma_{-1}(n-n_0) + A\frac{\alpha}{2} \left[\gamma_{-1}(n-n_0 + \frac{2\pi}{\Omega_0}) + \gamma_{-1}(n-n_0 - \frac{2\pi}{\Omega_0})\right].$$

*Sketch* of such a step response:

![Step Response Sketch](image-url)
Idealized Linear, Shift-invariant Systems

Attenuation Distortions – Part 21

**Idealized attenuation ripples (continued):**

Final **Remarks:**

- Each input signal $v(n)$ is reproduced in such a system **without distortions** at the output (neglecting the delay and the constant gain). In addition to that, the input signal **appears once before and once after the main signal** with a time shift of $2\pi/\Omega_0$. This time shift is proportional to the inverse of the ripple frequency.

- If $\alpha$ is large the echoes will be **audible in HiFi applications** – especially the „pre-echo“.
Some Questions

**Idealized attenuation ripples (continued):**

Partner work – Please think about the following questions and try to find answers (first group discussions, afterwards broad discussion in the whole group).

- If you design an equalization filter for a loudspeaker-amplifier system, what might be adequate cost functions that you could use in order to evaluate the “performance” of the equalization?

- Why is (in speech and audio applications) a “pre-echo” more disturbing than a “post-echo”? 
Real-valued systems without group-delay distortions:

If we assume a system with a linear phase, we can write for the frequency response

$$H(e^{j\Omega}) = H_0(\Omega) e^{-j\Omega n_0}.$$  

If we assume in addition that the corresponding impulse response should be real

$$h_0(n) = \mathcal{F}^{-1}\{H(e^{j\Omega})\} \in \mathbb{R},$$

we obtain – as known from the first parts of this lecture – that we get a symmetry in the frequency domain:

$$H_0(\Omega) = H_0^*(-\Omega) \in \mathbb{C}.$$  

The linear phase term $e^{-j\Omega n_0}$ needs not to be mentioned, because it results only in a temporal shift of the impulse response.
Idealized Linear, Shift-invariant Systems

Attenuation Distortions – Part 23

**Real-valued systems without group-delay distortions (continued):**

The phase contribution of the term $H_0(\Omega)$ can be described by

$$\arg\{H_0(\Omega)\} = \arctan\left(\frac{\text{Im}\{H_0(\Omega)\}}{\text{Re}\{H_0(\Omega)\}}\right).$$

Since the overall phase should be linear, the phase contribution $\arg\{H_0(\Omega)\}$ should not influence the overall phase. Thus, the phase $\arg\{H_0(\Omega)\}$ has **two options:**

- $\arg\{H_0(\Omega)\} = 0,$
- $\arg\{H_0(\Omega)\} = \frac{\pi}{2} \text{sign}(\Omega).$
Real-valued systems without group-delay distortions (continued):

Since the symmetry

\[ H_0(\Omega) = H_0^*(-\Omega) \]

has to be fulfilled, the first solution, \( \arg\{H_0(\Omega)\} = 0 \), leads to a real, even function in \( \Omega \):

\[ H_0(\Omega) \in \mathbb{R}, \text{ even concerning } \Omega. \]

As a result, the inverse Fourier transform (see first part of the lecture) leads to a real, even sequence concerning the time index:

\[ \mathcal{F}^{-1}\{H_0(\Omega)\} \in \mathbb{R}, \text{ even concerning } n. \]

If we also consider the additional phase term that leads to a shift in time we obtain finally:

\[ \mathcal{F}^{-1}\{H_0(\Omega) e^{-j\Omega n_0}\} = h_0(n) \in \mathbb{R}, \text{ even concerning } (n - n_0). \]
Real-valued systems without group-delay distortions (continued):

The impulse response of the first solution is real and even-symmetric concerning $n_0$. Thus, we have:

$$h_0(n_0 - n) = h_0(n_0 + n).$$

For the second solution we assumed that $H_0(\Omega)$ is imaginary and even concerning $\Omega$. In that case we obtain for the inverse Fourier transform of the term $H_0(\Omega)$:

$$\mathcal{F}^{-1}\{H_0(\Omega)\} \in \mathbb{R}, \text{ odd concerning } n.$$

If we also consider the linear phase term (shift in the time domain) we obtain for the symmetry of the resulting impulse response:

$$h_0(n_0 - n) = -h_0(n_0 + n).$$
Idealized Linear, Shift-invariant Systems

Attenuation Distortions – Part 26

Real-valued systems without group-delay distortions (continued):

Please find four examples for the four different types of impulse responses that result in a linear phase and thus in a constant group delay.

In addition to the even and odd symmetry we differentiate also between even and odd filter orders.

![Graphs showing even and odd filter lengths for cases 1 to 4.](image)
**Basics:**

Finally, we assume that a filter should have a constant magnitude frequency response:

$$|H(e^{j\Omega})| = C, \ \forall \Omega.$$  

If the phase of the filter is not linear (or has a sign-function based phase), meaning that we have

$$\arg\{H(e^{j\Omega})\} \neq \phi_0 \text{sign}(\Omega) + n_0 \Omega,$$

then we talk about all-pass filters with non-linear phases. If we transform such frequency responses back to the time domain, we obtain:

$$h_0(n) = \mathcal{F}^{-1}\{H(e^{j\Omega})\}$$

... inserting the definition of the frequency response ...

$$= \frac{1}{2\pi} \int_{\Omega = -\pi}^{\pi} C e^{-j\arg\{H(e^{j\Omega})\}} e^{j\Omega n} d\Omega$$

... inserting $e^{jx} = \cos(x) + j\sin(x)$ ...

$$= \frac{C}{2\pi} \left[ \int_{\Omega = -\pi}^{\pi} \cos\left(\Omega n - \arg\{H(e^{j\Omega})\}\right) d\Omega + j \int_{\Omega = -\pi}^{\pi} \sin\left(\Omega n - \arg\{H(e^{j\Omega})\}\right) d\Omega \right]$$
Idealized Linear, Shift-invariant Systems

Phase Distortions – Part 2

*Basics (continued):*

Inverse Fourier transform (continued):

\[
h_0(n) = \frac{C}{2\pi} \left[ \int_{\Omega = -\pi}^{\pi} \cos \left( \Omega n - \arg \{ H(e^{i\Omega}) \} \right) \, d\Omega + j \int_{\Omega = -\pi}^{\pi} \sin \left( \Omega n - \arg \{ H(e^{i\Omega}) \} \right) \, d\Omega \right].
\]

Since we do not have any restrictions on the phase response we will produce *no symmetries in the impulse response* (and thus also not in the step response).
Idealized Linear, Shift-invariant Systems

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Next part:
- Hilbert transform