Christian-Albrechts-Universität zu Kiel

Advanced Signals and Systems

Part 1: Introduction

Gerhard Schmidt

Christian-Albrechts-Universität zu Kiel
Faculty of Engineering
Institute of Electrical Engineering and Information Engineering
Digital Signal Processing and System Theory

Contents of the Lecture

Today:

- Boundary conditions of the lecture
  - Contents
  - Literature hints
  - Exams
- Start with first part (discrete signals and processes) of the lecture

Contents of the Lecture

Entire Semester:

- Introduction
- Discrete signals and random processes
- Spectra
- Discrete systems
- Idealized linear, shift-invariant systems
- Hilbert transform
- State-space description and system realizations
- Generalizations for signals, systems, and spectra

English (and German) Books:

Statistical signal theory:

- A. Papoulis: ***Probability, Random Variables, and Stochastic Processes***, McGraw Hill, 1965
- E. Hänsler: ***Statistische Signale: Grundlagen und Anwendungen***, Springer, 2001 (in German)

Discrete signal processing:

- A. V. Oppenheim, R. W. Schafer: ***Discrete-Time Signal Processing***, 2nd edition, Prentice Hall, 1999
- J. G. Proakis, D. K. Manolakis: ***Digital Signal Processing***, 4th edition, Prentice Hall, 2006
- K. D. Kammeyer, K. Kroschel: ***Digitale Signalverarbeitung - Filterung und Spektralanalyse mit MATLAB-Übungen***, Teubner, 2002 (in German)

Signal processing:

- A. V. Oppenheim, A. S. Willsky, S. Hamid: ***Signals and Systems***, 2nd edition, Prentice Hall, 1996
- S. Haykin, B. Van Veen: ***Signals and Systems***, 2nd edition, Wiley, 2002

Boundary Condition of the Lecture

Credit Points, Exams, Exercises, and Lecture Notes

Credit points:

- 7 ECTS points

Written exam:

- 90 minutes test
- In the exams period

Exercises:

- Every week two hours (45 min) during the semester

Lecture notes:

- Printed versions will be spread at the beginning of each lecture
- In the internet as pdf files via ***www.dss.tf.uni-kiel.de***

Origin of this Lecture

Thanks to …

***… Prof. Dr.-Ing. Ulrich Heute
(slides are based on his script)***

Prof. Heute has given this lecture until 2010.

The DSS Team

People that you can ask for help

- The DSS team has no "question hours" … just come over, someone will have time for you.
- Details (where we are located, etc.) can be found via ***www.dss.tf.uni-kiel.de***.

Exercises

- M.Sc. Anne Theiß

Some General Words about Signals and Systems

"Advanced" Signals and Systems:

We will treat the basic theory of ***continuous, deterministic, one-dimensional signals***. We assume that ***basic knowledge about continuous, stochastic, one-dimensional signals*** as well as theory of ***continuous, one-dimensional systems with one input and one output are known***. What we will treat here are:

- ***Signals*** (discrete signals, sequences) representing (in an abstract manner) any entity depending on any independent variable (e.g. pixel brightness on a 2-D grid). Please note that we call signals to be ***digital***, if a discrete signal has also a discrete (quantized) amplitude.
- ***Systems*** are ***operators*** that are excited by input sequences, creating internal (state) sequences and output sequences. ***Digital systems*** are operators that are working on ***digital inputs*** with ***digital parameters*** (e.g. quantized coefficients) and quantized states yielding ***digital outputs***.

In the following …

… we will restrict ourselves ***to discrete signals and systems*** with a ***short extension*** towards ***digital signals*** and some references to ***continuous (analog) signals and systems***.

Scalars and Vectors

Scalars:

Discrete time index

- Signals: $v(n)$

Coefficient index

- Impulse responses (time-variant): $h_i(n)$
- Example for a (real) convolution: $y(n) = \sum_{i=0}^{N-1} v(n-i)\, h_i(n)$

Vectors:

Boldface and lowercase

- Signal vectors: $\boldsymbol{v}(n) = \big[v(n),\, v(n-1),\, \dots,\, v(n-N+1)\big]^{\mathrm{T}}$
- Impulse response vectors (time-variant) : $\boldsymbol{h}(n) = \big[h_0(n),\, h_1(n),\, \dots,\, h_{N-1}(n)\big]^{\mathrm{T}}$
- Example for a real convolution: $y(n) = \boldsymbol{v}^{\mathrm{T}}(n)\,\boldsymbol{h}(n) = \boldsymbol{h}^{\mathrm{T}}(n)\,\boldsymbol{v}(n)$

Matrices:

$$\boldsymbol{A}(n) = \begin{bmatrix} a_{00}(n) & a_{01}(n) & \dots & a_{0N}(n) \\ a_{10}(n) & a_{11}(n) & \dots & a_{1N}(n) \\ \vdots & \vdots & & \vdots \\ a_{M0}(n) & a_{M1}(n) & \dots & a_{MN}(n) \end{bmatrix}$$

Boldface and uppercase

Signals (Details):

- Notation:

$$\{v(n),\, n \in \mathbb{Z}\} \quad \stackrel{\text{sloppy}}{\longrightarrow} \quad v(n)$$

with

$$v(n) \in \mathbb{C}.$$

Non-quantized, complex quantity!

- Signal vector versus M –dimensional signals:

E.g. speed of an object (x, y, and z-direction)

$$\boldsymbol{v}(n)$$

$$v(\boldsymbol{n}) \;=\; v(n_0,\, n_1,\, \ldots,\, n_{M-1})$$

E.g. brightness of a picture (M=2)

During the lecture we will focus on one-dimensional scalar signals!

Random variables and processes:

- Notation: $x(n),\ x_1(n),\ x_2(n)$

No differences between deterministic signals and random processes – different writing styles: $\boldsymbol{x}(\eta, n),\ x(\omega, n),\ X(n)$

- Probability density function: $f_x(x, n),\ f_{x_1 x_2}(x_1, x_2, n_1, n_2)$

- Stationary random processes:

$$\begin{aligned} f_x(x,\, n) &= f_x(x,\, n+n_0) = f_x(x) \\ f_{x_1x_2}(x_1,\, x_2,\, n_1,\, n_2) &= f_{x_1x_2}(x_2,\, x_2,\, n_1+n_0,\, n_2+n_0) \\ &= f_{x_1x_2}(x_1,\, x_2,\, n_2-n_1) \end{aligned}$$

- Expected values of stationary random processes:

$$\mathrm{E}\Big\{x(n)\Big\} = \int\limits_{x=-\infty}^{\infty} x\, f_x(x)\, dx = \mu_x^{(1)} = \mu_x$$

$$\mathrm{E}\Big\{x^2(n)\Big\} = \int\limits_{x=-\infty}^{\infty} x^2\, f_x(x)\, dx = \mu_x^{(2)}, \qquad \mathrm{E}\Big\{g\big(x(n)\big)\Big\} = \int\limits_{x=-\infty}^{\infty} g(x)\, f_x(x)\, dx$$

Correlation

Auto and cross correlation for real, stationary random processes:

- Auto-correlation function:

$$\mathrm{E}\Big\{v(n)\,v(n+l)\Big\} \;=\; s_{vv}(l)$$

- Cross-correlation function:

$$\mathrm{E}\Big\{v(n)\,y(n+l)\Big\} \;=\; s_{vy}(l)$$

- (Auto) power spectral density:

$$S_{vv}(\Omega) \;=\; \sum_{l=-\infty}^{\infty} \mathrm{E}\Big\{v(n)\,v(n+l)\Big\}\,e^{-j\Omega\,l} \;=\; \sum_{l=-\infty}^{\infty} s_{vv}(l)\,e^{-j\Omega\,l}$$

- (Cross) power spectral density:

$$S_{vy}(\Omega) \;=\; \sum_{l=-\infty}^{\infty} \mathrm{E}\Big\{v(n)\,y(n+l)\Big\}\,e^{-j\Omega\,l} \;=\; \sum_{l=-\infty}^{\infty} s_{vy}(l)\,e^{-j\Omega\,l}$$

White Noise

Stationary white noise:

- Auto-correlation function:

$$s_{vv}(l)\Big|_{\text{white noise}} = \begin{cases} \sigma_v^2, & \text{if } l = 0, \\ 0, & \text{else.} \end{cases}$$

- Auto power spectral density:

$$S_{vv}(\Omega)\Big|_{\text{white noise}} = \sigma_v^2$$

A First Exercise

Please try on your own:

- A linear, causal, and (time-) shift-invariant system – specified by its impulse response h_i – is excited with zero-mean white noise $v(n)$ with variance σ_v^2. What is the output power of the system?

Summary and Outlook

This part:

- Boundary conditions of the lecture
  - Contents
  - Literature hints
  - Exams, credit points, etc.
- Notation

Next part:

- Discrete signals and random processes