

Advanced Signals and Systems

Exam SS 2018

Examiner: Prof. Dr.-Ing. Gerhard Schmidt

Date: 27.09.2018

Name: _____

Matriculation Number: _____

Declaration of the candidate before the start of the examination

I hereby confirm that I am registered for, authorized to sit and eligible to take this examination.

I understand that the date for inspecting the examination will be announced by the EE&IT Examination Office, as soon as my provisional examination result has been published in the QIS portal. After the inspection date, I am able to request my final grade in the QIS portal. I am able to appeal against this examination procedure until the end of the period for academic appeals for the second examination period at the CAU. After this, my grade becomes final.

Signature: _____

Marking

Problem	1	2	3
Points	/33	/33	/34

Total number of points: _____ /100

Inspection/Return

I hereby confirm that I have acknowledged the marking of this examination and that I agree with the marking noted on this cover sheet.

- The examination papers will remain with me. Any later objection to the marking or grading is no longer possible.

Kiel, dated _____ Signature: _____

Advanced Signals and Systems

Exam SS 2018

Examiner: Prof. Dr.-Ing. Gerhard Schmidt
Date: 27.09.2018
Time: 09:00 h – 10:30 h (90 minutes)
Location: KS2/Geb.C - SR I

Remarks

- Please write your **name** and your **matriculation number** on each sheet of paper that you return.
- Please keep your student ID and your identity card ready.
- You may not start working on the exam until you are specifically told to do so.
- During the exam only questions concerning the problems are answered.
- Please use a **new** sheet of paper with your name and matriculation number on it for **each problem**. You can ask for more sheets of paper, if necessary.
- Please don't use any pencil or red pen.
- The exam is open books, open notes; other people are closed. All electronic devices except non-programmable pocket calculators are prohibited.
- Partial credit will be given. No credit will be given if an answer appears with no supporting work or reason. All axes in sketches must be labeled to receive full credit.
- Note that the given points of the subproblems are just preliminary.
- You will be informed about the approaching end of the exam. This will be done orally five and one minute prior to the end of the exam. Once the end of the exam has been announced, you must **stop writing immediately**.
- In case you should feel negatively impacted by your surroundings during the exam, you must notify the exam supervisor immediately.
- At the end of the exam put all sheets together as you have received them, including the problem sheets and the **sheet** with your **signature** on it.
- No one is allowed to talk or to leave his or her seat until **all** exams have been collected. Any talking during this time may be considered an **attempt of deception**.
- The problems and the solutions will be published on the website of the lecture. Also the date and the place of the inspection will be announced on this website.

Problem 1 (33 points)

Part 1 This part may be solved independently of parts 2 and 3.

Given is the probability density function of a real stochastic variable x with real constants $\alpha, \beta > 0$ and Euler's constant e :

$$f_x(x) = \frac{1}{\alpha} e^{-\frac{|x|}{\beta}}.$$

(a) Give an expression for β in terms of α . Give reason to your answer! (2 P)
 The density function belongs to a Laplace distribution. From this follows $\beta = \frac{\alpha}{2}$.

(b) Determine the probability function $F_x(x)$ as a function of α and β . (5 P)
 The following generally holds:

$$F_x(x) = \int_{-\infty}^x f_x(t) dt$$

Therefore, the distribution function may be computed as:

$$F_x(x) = \int_{-\infty}^x \frac{1}{\alpha} e^{-\frac{|t|}{\beta}} dt$$

$x \leq 0$:

$$F_x(x) = \int_{-\infty}^x \frac{1}{\alpha} e^{\frac{t}{\beta}} dt = \frac{1}{\alpha} \left[\beta e^{\frac{t}{\beta}} \right]_{-\infty}^x = \frac{\beta}{\alpha} e^{\frac{x}{\beta}}$$

$x > 0$:

$$F_x(x) = \int_{-\infty}^0 \frac{1}{\alpha} e^{\frac{t}{\beta}} dt + \int_0^x \frac{1}{\alpha} e^{-\frac{t}{\beta}} dt = \frac{\beta}{\alpha} + \frac{1}{\alpha} \left[-\beta e^{-\frac{t}{\beta}} \right]_0^x = 2\frac{\beta}{\alpha} - \frac{\beta}{\alpha} e^{-\frac{x}{\beta}} = \frac{\beta}{\alpha} \left(2 - e^{-\frac{x}{\beta}} \right)$$

(c) How may your result from (a) be validated using your result from (b)? (2 P)
 The limit of the distribution function must be identical 1 for $x \rightarrow \infty$. By inserting the result from (a) into the expression of the limit, the results may be checked:

$$\lim_{x \rightarrow \infty} F_x(x) = 2\frac{\beta}{\alpha}$$

$$2\frac{\beta}{\alpha} \Big|_{\beta=\frac{\alpha}{2}} = 1$$

Now consider the following deterministic transform of the above stochastic variable:

$$\gamma = 2e^x + 7$$

(d) Determine the corresponding density function $f_\gamma(\gamma)$. (4 P)

Using the transform $g(x) = 2e^x + 7$ and its inverse $h(\gamma) = \ln\left(\frac{\gamma-7}{2}\right)$ for $\gamma > 7$:

$$f_\gamma(\gamma) = \begin{cases} f_x(h(\gamma)) |h'(\gamma)| = \frac{1}{\alpha} e^{-\frac{|\ln(\frac{\gamma-7}{2})|}{\beta}} \cdot \left| \frac{1}{\gamma-7} \right|, & \text{for } \gamma > 7, \\ 0, & \text{else.} \end{cases}$$

Part 2 This part may be solved independently of parts 1 and 3.

Given is the real stochastic variable δ , which properties are fully described by its uniform distribution over the interval $[-\frac{\pi}{2}, \frac{\pi}{2})$. Additionally consider a complex deterministic transform of the variable:

$$\nu_1 = e^{j\delta} + 2$$

(e) Give both the probability density function f_δ and the probability function F_δ . (3 P)

$$f_\delta(\delta) = \begin{cases} \frac{1}{\pi}, & \text{for } -\frac{\pi}{2} \leq \delta < \frac{\pi}{2} \\ 0, & \text{else} \end{cases}$$

$$F_\delta(\delta) = \begin{cases} 0, & \text{for } \delta < -\frac{\pi}{2} \\ \frac{1}{\pi}\delta + \frac{1}{2}, & \text{for } -\frac{\pi}{2} \leq \delta < \frac{\pi}{2} \\ 1, & \text{else} \end{cases}$$

(f) Determine the expected value of ν_1 independently for its real and imaginary part. (4 P)
The variable may be split into $\text{Re}\{\nu_1\} = \cos(\delta) + 2$ and $\text{Im}\{\nu_1\} = \sin(\delta)$. Hence:

$$\text{E}\{\text{Re}\{\nu_1\}\} = \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos(\delta) + 2) d\delta = \frac{1}{\pi} \left[\sin(\delta) + 2\delta \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{1}{\pi} (1 + \pi + 1 + \pi) = 2 + \frac{2}{\pi}$$

$$\text{E}\{\text{Im}\{\nu_1\}\} = \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin(\delta) d\delta = \frac{1}{\pi} \left[-\cos(\delta) \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 0$$

Now consider a modified transform $\nu_2 = e^{j(\delta_1 + \delta_2)}$ and the following joint density function:

$$f_{\delta_1, \delta_2}(\delta_1, \delta_2) = \begin{cases} 0, & \text{for } \delta_1 < 0 \vee \delta_2 < 0 \vee \delta_1 \geq \pi \vee \delta_2 \geq \pi \\ \frac{2}{\pi^2} - \frac{2}{\pi^3}\delta_1, & \text{else.} \end{cases}$$

(g) Determine the expected value of the real part of ν_2 . (5 P)

Hint: $\int x \sin(x) dx = \sin(x) - x \cos(x) + C$.

It holds:

$$\begin{aligned} \nu_2 &= e^{j(\delta_1 + \delta_2)} = (\cos(\delta_1) + j \sin(\delta_1)) (\cos(\delta_2) + j \sin(\delta_2)) \\ &= \cos(\delta_1) \cos(\delta_2) - \sin(\delta_1) \sin(\delta_2) + j \text{Im}\{\nu_2\} \end{aligned}$$

$$\begin{aligned} \text{E}\{\text{Re}\{\nu_2\}\} &= \int_0^\pi \left(\frac{2}{\pi^2} - \frac{2}{\pi^3}\delta_1 \right) \cos(\delta_1) \int_0^\pi \cos(\delta_2) d\delta_2 d\delta_1 \\ &\quad - \int_0^\pi \left(\frac{2}{\pi^2} - \frac{2}{\pi^3}\delta_1 \right) \sin(\delta_1) \int_0^\pi \sin(\delta_2) d\delta_2 d\delta_1 \\ &= 0 - 2 \int_0^\pi \left(\frac{2}{\pi^2} - \frac{2}{\pi^3}\delta_1 \right) \sin(\delta_1) d\delta_1 \\ &= \frac{4}{\pi^3} \int_0^\pi \delta_1 \sin(\delta_1) d\delta_1 - \frac{4}{\pi^2} \int_0^\pi \sin(\delta_1) d\delta_1 \\ &= \frac{4}{\pi^3} \left[\sin(\delta_1) - \delta_1 \cos(\delta_1) \right]_0^\pi - \frac{8}{\pi^2} \\ &= \frac{4\pi}{\pi^3} - \frac{8}{\pi^2} = -\frac{4}{\pi^2} \end{aligned}$$

Part 3 This part may be solved independently of parts 1 and 2.

Assume a real and uncorrelated stochastic random sequence $z(n)$. The variables m_z and σ_z are expected value and standard deviation of the corresponding ergodic random process, respectively.

(h) Show that the following holds for the autocorrelation function: (4 P)

$$s_{zz}(\kappa) = m_z^2 + \sigma_z^2 \cdot \gamma_0(\kappa).$$

The following sketches a possible solution. For this, the sequences $z_1(n)$ and $z_2(n)$ are defined, which possess the same stochastic properties as $z(n)$ but are uncorrelated with each other.

$$\begin{aligned} s_{zz}(\kappa) &= \mathbb{E}\{z(n) \cdot z(n + \kappa)\} \\ &= \begin{cases} \mathbb{E}\{z^2(n)\}, & \text{für } \kappa = 0 \\ \mathbb{E}\{z_1(n) \cdot z_2(n)\}, & \text{für } \kappa \neq 0 \end{cases} \\ &= \begin{cases} \mathbb{E}\{z^2(n)\} - \mathbb{E}^2\{z(n)\} + \mathbb{E}^2\{z(n)\}, & \text{für } \kappa = 0 \\ \mathbb{E}\{z_1(n)\} \cdot \mathbb{E}\{z_2(n)\}, & \text{für } \kappa \neq 0 \end{cases} \\ &= \begin{cases} \mathbb{E}\{(z(n) - m_z)^2\} + m_z^2, & \text{für } \kappa = 0 \\ \mathbb{E}\{z(n)\} \cdot \mathbb{E}\{z(n)\}, & \text{für } \kappa \neq 0 \end{cases} \\ &= \begin{cases} \sigma_z^2 + m_z^2, & \text{für } \kappa = 0 \\ m_z^2, & \text{für } \kappa \neq 0 \end{cases} \\ &= m_z^2 + \sigma_z^2 \cdot \gamma_0(\kappa) \end{aligned}$$

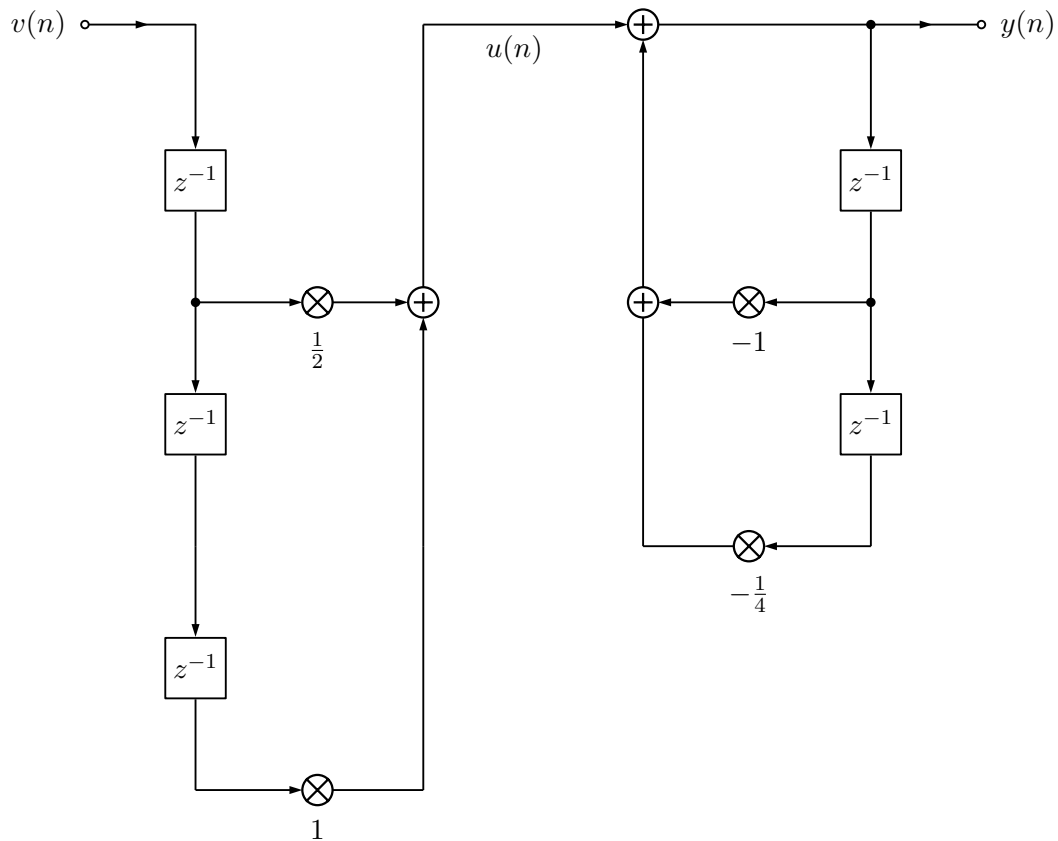
(i) Explain in what ways the properties of above sequence at $m_z = 0$ are practical for system identification (estimation of an impulse response) purposes. Give the necessary steps of the applied estimation method. (4 P)

For estimating an impulse response, the transfer function in the frequency domain may be estimated using power spectral densities (PSD). For this, the auto PSD of the input $z(n)$ and either the auto PSD of the output (estimation without phase) or the cross PSD between input and output signals (estimation with phase) need to be calculated. By rearranging the known formula, the transfer function may then be computed. The signal $z(n)$ is practical here, since it is zero-mean white noise which avoids zero-divisions. Additionally, the frequency-selective division breaks down into a scaling of the entire spectrum. Alternatively, the impulse response may be directly computed in the time domain, where the computation breaks down into a scaling of the cross correlation function between input and output signal.

Problem 2 (33 points)

Part 1 This problem may be solved independently of part 2.

The following blockdiagram is given. All memory-elements are initialized with 0 for $n < 0$.



In addition the input signal is defined as follows:

$$v(n) = [\gamma_{-1}(n+2) - \gamma_{-1}(n-2)] \cdot \gamma_0(n-3) + \gamma_0(n) + \gamma_{-1}(n-2) - 2\gamma_{-1}(n-4) + 3\gamma_0(n-5) + \gamma_{-1}(n-7) + \gamma_0(n-8) \cdot \gamma_{-1}(n-9).$$

- (a) Which kind of digital filter implementation structure is used in the block diagram shown above? (Name it!) (1 P)

Directform I.

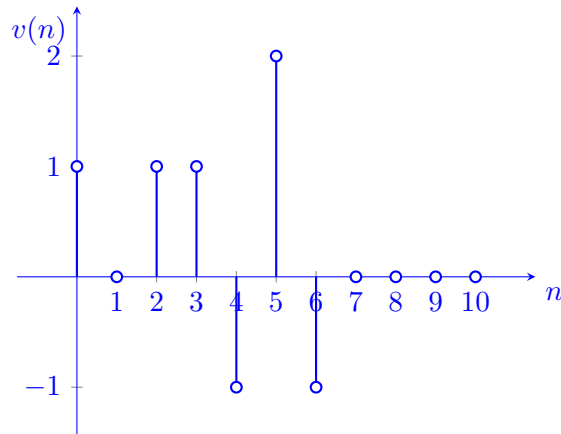
- (b) Sketch $v(n)$ for $0 \leq n < 11$. (2 P)

$v(n)$ can be simplified due to the selective property of the impulse function $\gamma_0(n)$:

$$v(n) = \gamma_0(n) + \gamma_{-1}(n-2) - 2\gamma_{-1}(n-4) + 3\gamma_0(n-5) + \gamma_{-1}(n-7)$$

$\mathbf{v} = [v(0), v(1), v(2), \dots, v(10)]^T$ for $0 \leq n < 11$ yields the following values:

$$\mathbf{v} = [1, 0, 1, 1, -1, 2, -1, 0, 0, 0, 0]^T$$



- (c) Determine the sequence $u(n)$ under consideration of the values of $v(n)$ for discrete time steps $0 \leq n < 11$. (3 P)

With \mathbf{v} and:

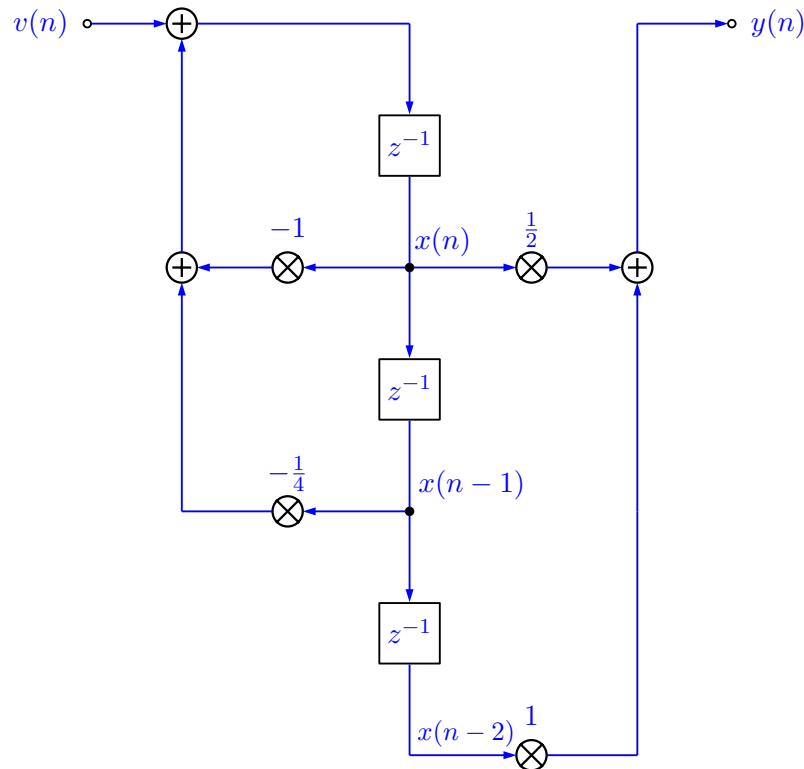
$$u(n) = \frac{1}{2} v(n-1) + v(n-3),$$

it follows $\mathbf{u} = [u(0), u(1), u(2), \dots, u(10)]^T$:

$$\mathbf{u} = \left[0, \frac{1}{2}, 0, \frac{3}{2}, \frac{1}{2}, \frac{1}{2}, 2, -\frac{3}{2}, \frac{1}{2}, -1, 0 \right]^T.$$

- (d) Sketch the direct-form II implementation and state an advantage of this implementation compared to the above given implementation. (4 P)

direct-form II requires less memory elements compared to direct-form I (in this case three) and is given by:



(e) How many states does the system have? (1 P)

Mark and name them in your solution of part (d).

Three states (see solution of previous part, $x(n), x(n-1), x(n-2)$).

(f) Determine the impulse response of the whole system. (7 P)

$$y(n) = u(n) - y(n-1) - \frac{1}{4}y(n-2)$$

$$y(n) + y(n-1) + \frac{1}{4}y(n-2) = \frac{1}{2}v(n-1) + v(n-3)$$

In z-domain it follows:

$$\left(1 + z^{-1} + \frac{1}{4}z^{-2}\right) Y(z) = \left(\frac{1}{2}z^{-1} + z^{-3}\right) V(z).$$

And such:

$$\begin{aligned} H(z) &= \frac{Y(z)}{V(z)} = \frac{\frac{1}{2}z^{-1} + z^{-3}}{1 + z^{-1} + \frac{1}{4}z^{-2}} = \frac{\frac{1}{2}z + z^{-1}}{\left(z + \frac{1}{2}\right)^2} \\ &= -\frac{-\frac{1}{2}z}{\left(z + \frac{1}{2}\right)^2} - 2z^{-2} \frac{-\frac{1}{2}z}{\left(z + \frac{1}{2}\right)^2}. \end{aligned}$$

Using known correspondences of the z-Transform ($x(n) = na^n\gamma_{-1}(n)$, $X(z) = \frac{za}{(z-a)^2}$) it follows:

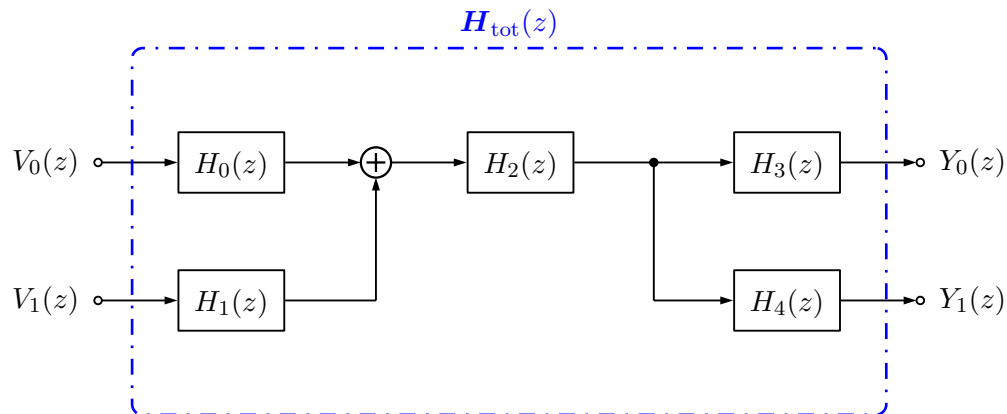
$$h(n) = -n \left(-\frac{1}{2}\right)^n \gamma_{-1}(n) - 2(n-2) \left(-\frac{1}{2}\right)^{(n-2)} \gamma_{-1}(n-2)$$

(g) Which part of the above given diagram has an FIR-characteristic and which part an IIR-characteristic? (Give reasons!) (3 P)

- **Part 1** with input $v(n)$ and output $u(n)$ (transversal structure) is of *FIR*-characteristic (no feedback of the output signal $u(n)$).
- **Part 2** with input $u(n)$ and output $y(n)$ (recursive structure) is of *IIR*-characteristic (feedback of the output signal $y(n)$).

Part 2 This part may be solved independently of part 1.

Now the following system is given:



(h) Determine $\mathbf{H}_{\text{tot}}(z)$ in dependence of $H_i(z)$, $i \in [0, 1, 2, 3, 4]$. What do the individual elements of $\mathbf{H}_{\text{tot}}(z)$ describe? (5 P)

$\mathbf{V}(z) = [V_0(z), V_1(z)]^T$ and $\mathbf{Y}(z) = [Y_0(z), Y_1(z)]^T$. Where $V_i(z)$ is the z-transform of $v_i(n)$ and $Y_j(z)$ those of $y_j(n)$. $i, j \in [0, 1]$.

$$\mathbf{Y}(z) = \mathbf{H}_{\text{tot}}(z) \mathbf{V}(z) = \begin{bmatrix} H_{0,0}(z) & H_{0,1}(z) \\ H_{1,0}(z) & H_{1,1}(z) \end{bmatrix} \cdot \begin{bmatrix} V_0(z) \\ V_1(z) \end{bmatrix}$$

$H_{i,j}$ describes the influence of input v_i to output y_j and is such given by:

$$\mathbf{H}_{\text{tot}}(z) = \begin{bmatrix} H_0(z)H_2(z)H_3(z) & H_1(z)H_2(z)H_3(z) \\ H_0(z)H_2(z)H_4(z) & H_1(z)H_2(z)H_4(z) \end{bmatrix}$$

Now solely the transfer path from $V_0(z)$ to $Y_0(z)$ is considered. In addition the following

definitions are given:

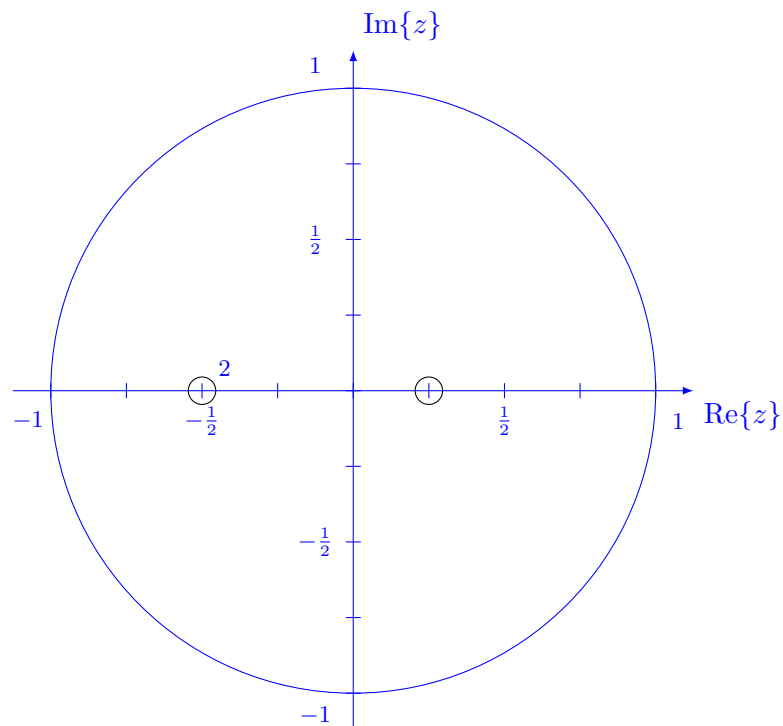
$$H_0(z) = \frac{\left(z + \frac{1}{2}\right)^3}{z - \frac{1}{4}}, \quad H_1(z) = \frac{z}{z^2 - 4}, \quad H_2(z) = \frac{z + \frac{1}{2}}{z^2 + \frac{1}{4}z - \frac{1}{8}},$$

$$H_3(z) = \frac{\left(z - \frac{1}{4}\right)^3}{z + \frac{1}{2}}, \quad H_4(z) = \frac{1}{\left(z - \frac{3}{4}\right)^3}.$$

(i) Sketch the pole/zero plot of the considered transfer path. (4 P)

$$\begin{aligned} H_{0,0}(z) &= H_0(z) H_2(z) H_3(z) \\ &= \frac{\left(z + \frac{1}{2}\right)^3}{z - \frac{1}{4}} \frac{z + \frac{1}{2}}{z^2 + \frac{1}{4}z - \frac{1}{8}} \frac{\left(z - \frac{1}{4}\right)^3}{z + \frac{1}{2}} \\ &= \frac{\left(z + \frac{1}{2}\right)^3}{z - \frac{1}{4}} \frac{z + \frac{1}{2}}{\left(z + \frac{1}{2}\right)\left(z - \frac{1}{4}\right)} \frac{\left(z - \frac{1}{4}\right)^3}{z + \frac{1}{2}} \\ &= \left(z + \frac{1}{2}\right)^2 \left(z - \frac{1}{4}\right) \end{aligned}$$

Two zeros at $z_{\infty,(1,2)} = -\frac{1}{2}$, one zero at $z_{\infty,3} = \frac{1}{4}$. This yields the following pole/zero plot:



(j) Is the considered subsystem: (3 P)

(i) stable,

Yes the system is stable as there are now poles outside the unit circle.

(ii) causal,

No as the the degree of the nominator ($M = 3$) is greater than those of the denominator ($N = 0$) and such the output $y(n)$ is dependent on future input values $v(n)$.

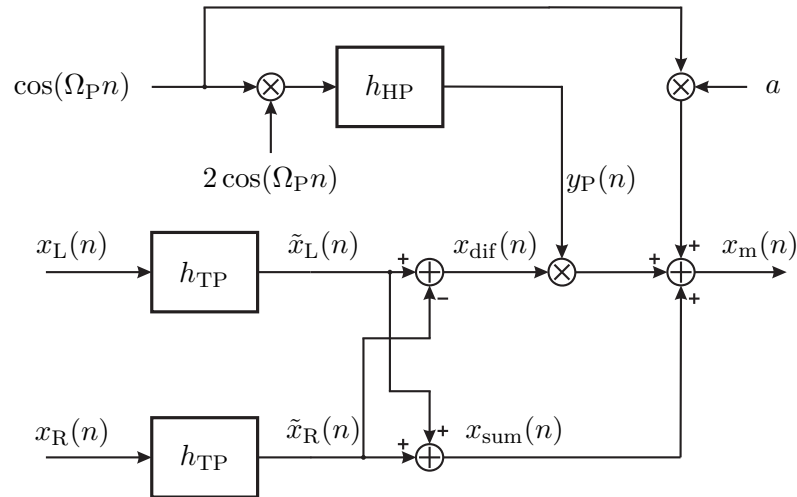
(iii) of minimal phase?

Yes as all zeros are within the unit circle.

Always give reason to your answers.

Problem 3 (34 points)

Given is a discrete system for the generation of a stereo baseband signal according to the following block diagram:



The filters h_{TP} and h_{HP} correspond to an ideal low-pass and an ideal high-pass filter, respectively. The cut-off frequencies of the two filters lie identically at Ω_C . In addition $\Omega_C < \Omega_P$ holds.

Simplify all of your solutions as far as possible using known theorems.

Part 1 This part may be solved independently of parts 2 and 3.

(a) Determine the signal $y_P(n)$. No convolution necessary! (3 P)

The ideal filter removes the DC component.

$$y_P(n) = [2 \cos(\Omega_P n) \cdot \cos(\Omega_P n)] * h_{HP}(n) = [1 + \cos(2\Omega_P n)] * h_{HP}(n) = \cos(2\Omega_P n).$$

(b) Determine the output signal $x_m(n)$ dependent on $\tilde{x}_L(n)$ and $\tilde{x}_R(n)$. (3 P)

$$x_m(n) = \tilde{x}_L(n) + \tilde{x}_R(n) + [\tilde{x}_L(n) - \tilde{x}_R(n)] \cos(2\Omega_P n) + a \cos(\Omega_P n)$$

(c) Determine the Fourier transform of your result from (b). (4 P)

$$\begin{aligned}
 X_m(e^{j\Omega}) &= \tilde{X}_L(e^{j\Omega}) + \tilde{X}_R(e^{j\Omega}) \\
 &+ \frac{1}{2\pi} \left[\tilde{X}_L(e^{j\Omega}) - \tilde{X}_R(e^{j\Omega}) \right] \otimes \left(\pi \sum_{k=-\infty}^{\infty} [\gamma_0(\Omega - 2\Omega_P - 2\pi k) + \gamma_0(\Omega + 2\Omega_P - 2\pi k)] \right) \\
 &+ a\pi \sum_{k=-\infty}^{\infty} [\gamma_0(\Omega - \Omega_P - 2\pi k) + \gamma_0(\Omega + \Omega_P - 2\pi k)] \\
 &= \tilde{X}_L(e^{j\Omega}) + \tilde{X}_R(e^{j\Omega}) + \frac{1}{2} \left[\tilde{X}_L(e^{j\Omega - 2\Omega_P}) + \tilde{X}_L(e^{j\Omega + 2\Omega_P}) \right] \\
 &- \frac{1}{2} \left[\tilde{X}_R(e^{j\Omega - 2\Omega_P}) + \tilde{X}_R(e^{j\Omega + 2\Omega_P}) \right] \\
 &+ a\pi \sum_{k=-\infty}^{\infty} [\gamma_0(\Omega - \Omega_P - 2\pi k) + \gamma_0(\Omega + \Omega_P - 2\pi k)]
 \end{aligned}$$

- (d) Determine the power spectral density $S_{x_{\text{dif}}x_{\text{dif}}}(e^{j\Omega})$ dependent on spectral densities (4 P) of the input signals. Assume that the signals $x_L(n)$ and $x_R(n)$ lie in the pass band range of the low-pass filter $h_{\text{TP}}(n)$.

$$\begin{aligned}
 s_{x_{\text{dif}}x_{\text{dif}}}(\kappa) &= \mathbb{E} \left\{ x_{\text{dif}}(n)x_{\text{dif}}^*(n + \kappa) \right\} \\
 &= \mathbb{E} \left\{ [x_L(n) - x_R(n)] [x_L(n + \kappa) - x_R(n + \kappa)]^* \right\} \\
 &= \mathbb{E} \left\{ x_L(n)x_L^*(n + \kappa) - x_R(n)x_L^*(n + \kappa) - x_L(n)x_R^*(n + \kappa) + x_R(n)x_R^*(n + \kappa) \right\}
 \end{aligned}$$

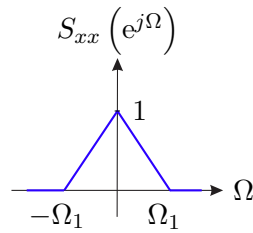
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$$\begin{aligned}
 S_{x_{\text{dif}}x_{\text{dif}}}(e^{j\Omega}) &= S_{x_Lx_L}(e^{j\Omega}) - S_{x_Rx_L}(e^{j\Omega}) - S_{x_Lx_R}(e^{j\Omega}) + S_{x_Rx_R}(e^{j\Omega}) \\
 &= S_{x_Lx_L}(e^{j\Omega}) - S_{x_Rx_L}(e^{j\Omega}) - S_{x_Rx_L}^*(e^{j\Omega}) + S_{x_Rx_R}(e^{j\Omega}) \\
 &= S_{x_Lx_L}(e^{j\Omega}) - 2\text{Re} \left\{ S_{x_Rx_L}(e^{j\Omega}) \right\} + S_{x_Rx_R}(e^{j\Omega})
 \end{aligned}$$

- (e) Is the power spectral density $S_{x_{\text{dif}}x_{\text{dif}}}(e^{j\Omega})$ complex or real? (1 P)

Auto power spectral densities are always real. Although the power density spectrum $S_{x_{\text{dif}}x_{\text{dif}}}(e^{j\Omega})$ contains two complex cross terms $-S_{x_Rx_L}(e^{j\Omega})$ and $-S_{x_Lx_R}(e^{j\Omega})$, these are complex conjugate to each other.

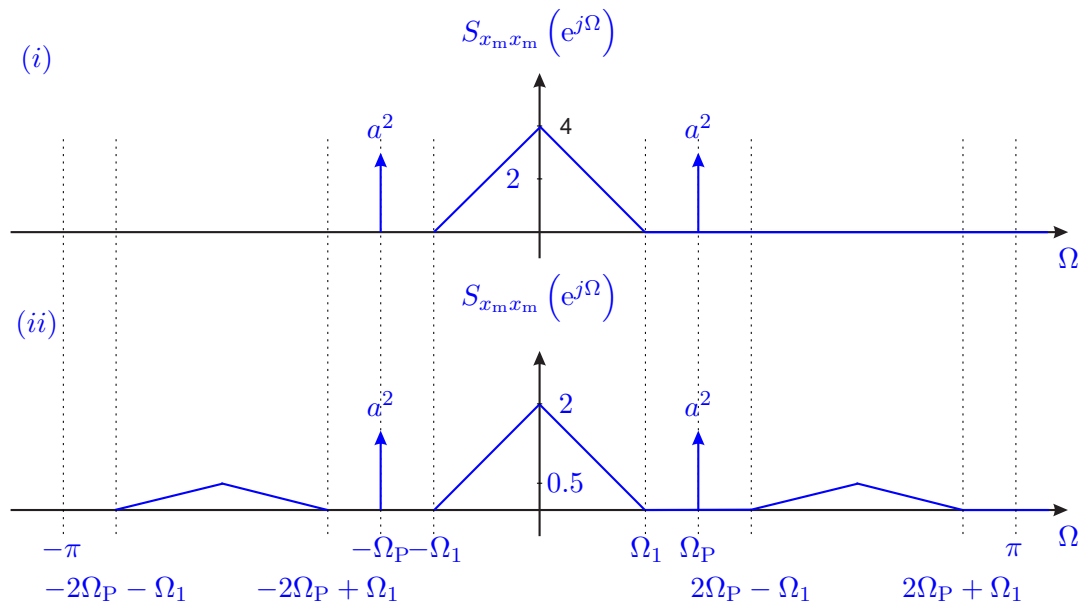
Now two processes $x_L(n)$ and $x_R(n)$ with the power spectral density $S_{xx}(e^{j\Omega}) = S_{x_Rx_R}(e^{j\Omega}) = S_{x_Lx_L}(e^{j\Omega})$ should be transmitted. In addition, $\Omega_1 < \Omega_C < \Omega_P$ should apply.



(f) Sketch the auto power spectral density $S_{x_m x_m}(e^{j\Omega})$ of the output process $x_m(n)$ for the following two cases. In addition, $3\Omega_P < \pi$ should apply.

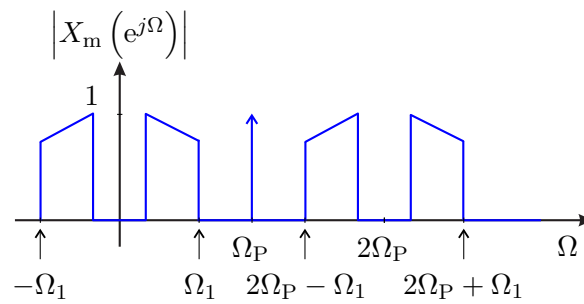
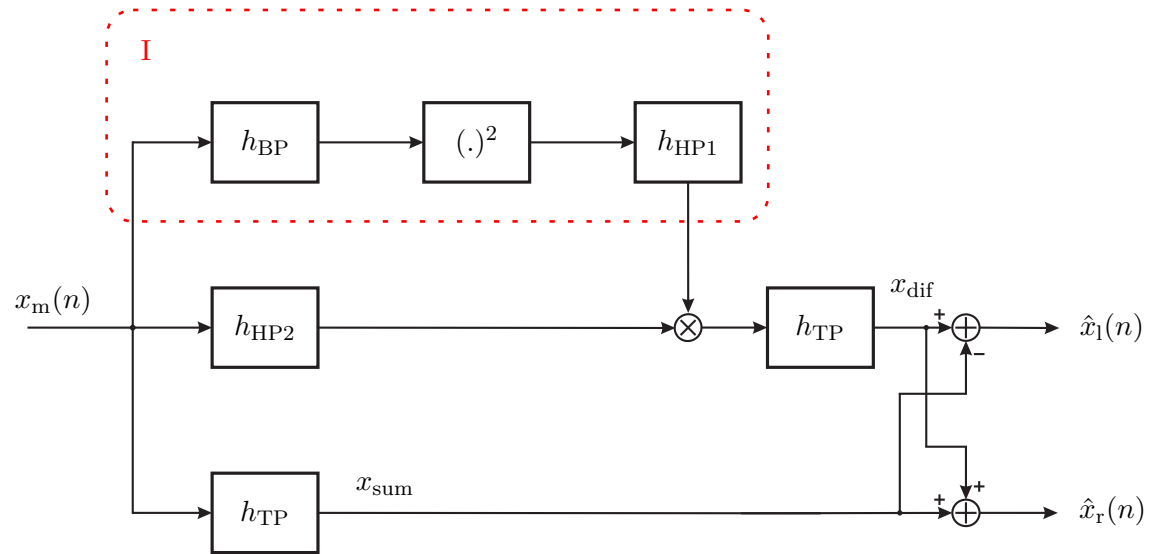
(i) The processes are identical, $x_R(n) = x_L(n)$. (3 P)

(ii) The processes $x_R(n)$ and $x_L(n)$ are orthogonal to each other. (4 P)



Part 2 This part may be solved independently of parts 1 and 3.

After a successful transmission and transformation into the baseband, a stereo baseband signal $x_m(n)$ is received and divided back into two stereo components $x_l(n)$ and $x_r(n)$ using the demodulator shown below. The magnitude spectrum of $x_m(n)$ is also given below.



(g) What is the purpose of the signal branch I? Explain in whole sentences. (2 P)

In the topmost branch I, the carrier is recovered. In the first step, the received signal $x_m(n)$ is filtered using a bandpass filter to isolate half the carrier frequency Ω_P . Subsequently, the DC component is removed with the high pass. At the end, a carrier signal with the frequency $2\Omega_P$ is created.

(h) Define the ideal filters h_{BP} , h_{HP1} , h_{HP2} and h_{TP} in the frequency domain for the range $0 < \Omega < \pi$. (8 P)

$$H_{BP}(e^{j\Omega}) = \begin{cases} 1, & \text{for } \Omega_1 < \Omega < 2\Omega_P - \Omega_1, \\ 0, & \text{else,} \end{cases}$$

$$H_{HP2}(e^{j\Omega}) = \begin{cases} 1, & \text{for } \Omega_P < \Omega < \pi, \\ 0, & \text{else,} \end{cases}$$

$$H_{HP1}(e^{j\Omega}) = \begin{cases} 1, & \text{for } \Omega_1 < \Omega < \pi, \\ 0, & \text{else,} \end{cases}$$

$$H_{TP}(e^{j\Omega}) = \begin{cases} 1, & \text{for } 0 < \Omega < \Omega_P, \\ 0, & \text{else.} \end{cases}$$

Part 3 This part may be solved independently of parts 1 and 2.

- (h) In the block diagram given in *part 1*, the difference of the two input signals $x_{\text{dif}}(n)$ (2 P) is multiplied by a carrier signal $y_{\text{P}}(n)$. Which modulation type does this implement? What type of modulation would you recommend to improve the efficiency of the transmission?

It is a double sideband amplitude modulation without carrier. The frequency efficiency can be improved by using single-sideband modulation. At the same time, the energy efficiency of transmission improves since the entire transmission energy is available to one sideband.

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