

**Problem 7** (FFT)

The  $M$ -point DFT of the  $M$ -point sequence  $x(n) = e^{-j(\pi/M)n^2}$ , for  $M$  even, is

$$X(\mu) = \sqrt{M} e^{-j\pi/4} e^{j(\pi/M)\mu^2}.$$

Determine the  $2M$ -point sequence  $y(n) = e^{-j(\pi/M)n^2}$ , assuming that  $M$  is even.

### Problem 8 (FFT of real and complex sequences)

Suppose that an FFT program is available that computes the DFT of a complex sequence. If we wish to compute the DFT of a real sequence, we may simply specify the imaginary part to be zero and use the program directly. However, the symmetry of the DFT of a real sequence can be used to reduce the amount of computation.

- (a) Let  $x(n)$  be a real-valued sequence of length  $M$ , and let  $X(\mu)$  be its DFT with real and imaginary parts denoted  $X_R(\mu)$  and  $X_I(\mu)$ , respectively; i.e.,

$$X(\mu) = X_R(\mu) + j X_I(\mu).$$

Show that if  $x(n)$  is real, then  $X_R(\mu) = X_R(M - \mu)$  and  $X_I(\mu) = -X_I(M - \mu)$  for  $\mu = 1, \dots, M - 1$ .

- (b) Now consider two real-valued sequences  $x_1(n)$  and  $x_2(n)$  with DFTs  $X_1(\mu)$  and  $X_2(\mu)$ , respectively. Let  $g(n)$  be the complex sequence  $g(n) = x_1(n) + j x_2(n)$ , with corresponding DFT  $G(\mu) = G_R(\mu) + j G_I(\mu)$ . Also, let  $G_{OR}(\mu)$ ,  $G_{ER}(\mu)$ ,  $G_{OI}(\mu)$  and  $G_{EI}(\mu)$  denote, respectively, the odd part of the real part, the even part of the real part, the odd part of the imaginary part, and the even part of the imaginary part of  $G(\mu)$ . Specifically, for  $1 \leq \mu \leq M - 1$ ,

$$G_{OR}(\mu) = 1/2\{G_R(\mu) - G_R(M - \mu)\},$$

$$G_{ER}(\mu) = 1/2\{G_R(\mu) + G_R(M - \mu)\},$$

$$G_{OI}(\mu) = 1/2\{G_I(\mu) - G_I(M - \mu)\},$$

$$G_{EI}(\mu) = 1/2\{G_I(\mu) + G_I(M - \mu)\},$$

and  $G_{OR}(0) = G_{OI}(0) = 0$ ,  $G_{ER}(0) = G_R(0)$ ,  $G_{EI}(0) = G_I(0)$ . Determine expressions for  $X_1(\mu)$  and  $X_2(\mu)$  in terms of  $G_{OR}(\mu)$ ,  $G_{ER}(\mu)$ ,  $G_{OI}(\mu)$  and  $G_{EI}(\mu)$ .