Problem 8 **(FFT)**

The M-point DFT of the M-point sequence $x(n) = e^{-j(\pi/M)n^2}$, for M even, is

$$X(\mu) = \sqrt{M}e^{-j\pi/4}e^{j(\pi/M)\mu^2}.$$

Determine the $2M$-point sequence $y(n) = e^{-j(\pi/M)n^2}$, assuming that M is even.

Problem 9 **(FFT of real and complex sequences)**

Suppose that an FFT program is available that computes the DFT of a complex sequence. If we wish to compute the DFT of a real sequence, we may simply specify the imaginary part to be zero and use the program directly. However, the symmetry of the DFT of a real sequence can be used to reduce the amount of computation.

(a) Let $x(n)$ be a real-valued sequence of length M, and let $X(\mu)$ be its DFT with real and imaginary parts denoted $X_R(\mu)$ and $X_I(\mu)$, respectively; i.e.,

$$X(\mu) = X_R(\mu) + j\,X_I(\mu).$$

Show that if $x(n)$ is real, then $X_R(\mu) = X_R(M-\mu)$ and $X_I(\mu) = -X_I(M-\mu)$ for $\mu = 1, ..., M-1$.

(b) Now consider two real-valued sequences $x_1(n)$ and $x_2(n)$ with DFTs $X_1(\mu)$ and $X_2(\mu)$, respectively. Let $g(n)$ be the complex sequence $g(n) = x_1(n) + j\,x_2(n)$, with corresponding DFT $G(\mu) = G_R(\mu) + j\,G_I(\mu)$. Also, let $G_{OR}(\mu)$, $G_{ER}(\mu)$, $G_{OI}(\mu)$ and $G_{EI}(\mu)$ denote, respectively, the odd part of the real part, the even part of the real part, the odd part of the imaginary part, and the even part of the imaginary part of $G(\mu)$. Specifically, for $1 \leq \mu \leq M-1$,

$$\begin{aligned}
G_{OR}(\mu) &= 1/2\{G_R(\mu) - G_R(M-\mu)\},\\
G_{ER}(\mu) &= 1/2\{G_R(\mu) + G_R(M-\mu)\},\\
G_{OI}(\mu) &= 1/2\{G_I(\mu) - G_I(M-\mu)\},\\
G_{EI}(\mu) &= 1/2\{G_I(\mu) + G_I(M-\mu)\},
\end{aligned}$$

and $G_{OR}(0) = G_{OI}(0) = 0$, $G_{ER}(0) = G_R(0)$, $G_{EI}(0) = G_I(0)$. Determine expressions for $X_1(\mu)$ and $X_2(\mu)$ in terms of $G_{OR}(\mu)$, $G_{ER}(\mu)$, $G_{OI}(\mu)$ and $G_{EI}(\mu)$.