

Problem 4 (DFT and convolution)

Let $h(n)$ be the sequence $\{1, 1, 0, 0, 0, 0, 0, 0\}$ and $y(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$.

- (a) Calculate the DFT of length 8 for both sequences.
- (b) Determine with help of the DFT a sequence $v(n)$ such that $y(n) = h(n) \otimes v(n)$.
- (c) Let $z(n)$ be the result of the linear convolution of $h(n)$ and $v(n)$: $z(n) = h(n) * v(n)$.
Is $z(n) = y(n)$?

Problem 5 (DFT)

The time-limited signal

$$v_0(t) = \begin{cases} \sin(\omega_0 t) & \text{for } 0 \leq t < 4\pi/\omega_0 \\ 0 & \text{otherwise} \end{cases}$$

is sampled with $t_n = nT_A = n\frac{\pi}{4\omega_0}$ to produce the time-limited sequence $v(n)$.

- (a) Sketch $v_0(t)$.
- (b) Determine $v(n)$.
- (c) Determine the DFT of $v(n)$.
- (d) Determine the Fourier Transform $V(e^{j\Omega})$ of $v(n)$.
- (e) Explain the connection between the DFT $\{v(n)\}$ and $V(e^{j\Omega})$.

Problem 6 (DFT, zero padding, leakage)

Let $v_a(t)$ be a time-continuous periodic signal

$$v_a(t) = 1 + \cos(2\pi 40t) + 3 \cdot \cos(2\pi 120t).$$

The signal is sampled ($\omega_s = 2\pi 280s^{-1}$) to produce the sequence $v(n)$. For practical purposes (delay, complexity) the sequence is limited to L samples. M is the length of the DFT. Use MATLAB to solve the following subproblems.

- (a) Sketch $v_a(t)$, $v(n)$, the Fourier transform $V(e^{j\Omega})$ and the DFT $V_M(\mu)$ for $L = 7$ and $M = 7$.
- (b) Sketch $v_a(t)$, $v(n)$, the Fourier transform $V(e^{j\Omega})$ and the DFT $V_M(\mu)$ for $L = 7$ and $M = 14$ (zero padding).
- (c) Sketch $v_a(t)$, $v(n)$, the Fourier transform $V(e^{j\Omega})$ and the DFT $V_M(\mu)$ for $L = 28$ and $M = 28$.
- (d) Sketch $v_a(t)$, $v(n)$, the Fourier transform $V(e^{j\Omega})$ and the DFT $V_M(\mu)$ for $L = 28$ and $M = 56$ (zero padding).
- (e) Sketch $v_a(t)$, $v(n)$, the Fourier transform $V(e^{j\Omega})$ and the DFT $V_M(\mu)$ for $L = 14$ and $M = 15$ (zero padding).
- (f) Sketch $v_a(t)$, $v(n)$, the Fourier transform $V(e^{j\Omega})$ and the DFT $V_M(\mu)$ for $L = 14$ and $M = 21$ (zero padding).
- (g) Sketch $v_a(t)$, $v(n)$, the Fourier transform $V(e^{j\Omega})$ and the DFT $V_M(\mu)$ for $L = 30$ and $M = 30$.
- (h) Sketch $v_a(t)$, $v(n)$, the Fourier transform $V(e^{j\Omega})$ and the DFT $V_M(\mu)$ for $L = 15$ and $M = 30$ (zero padding).