Problem 3 (Quantization)

A sinusoid signal $v(n) = 5\sin(\frac{\omega_0}{\omega_s} \cdot n)$ with $f_0 = 5$ Hz and $f_s = 10$ kHz has to be quantized ($v_q = Q[v(n)]$) with a midtreat quantizer. The range of the signal is ± 5 V and the word length of the quantizer 4 bits. The quantizer at digital full scale.

(a) How many quantization levels L does the quantizer have? What is the value of Δ?

(b) Sketch the input-output characteristic of the quantizer. How different is a midtreat quantizer to a midrise quantizer.

(c) For time index $n = 1250$ calculate the quantized value $v_q(n)$, the quantization error $e_q(n)$ and represent $v_q(n)$ using bipolar code (sign and magnitude representation).

The quantization error over time can be modeled as a noise that is added to the input signal.

(d) Sketch the real system and the mathematical model of the system with the added quantization noise.

(e) Calculate the power P_n of the quantization noise.

(f) Determine the SNR in dB and in linear scale.

The signal's amplitude is changed to ± 1 V, while the range R of the quantizer remains the same as before.

(g) How is SNR affected with this change?

(h) What word length has to be chosen to achieve an SNR > 45 dB?