Advanced Digital Signal Processing

Examination WS 2017/2018

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Date:	09.03.2018
Name:	______________________
Matriculation Number:	______________________

Declaration of the candidate before the start of the examination

I hereby confirm that I am registered for, authorised to sit and eligible to take this examination.
I understand that the date for inspecting the examination will be announced by the EE&IT Examination Office, as soon as my provisional examination result has been published in the QIS portal. After the inspection date, I am able to request my final grade in the QIS portal. I am able to appeal against this examination procedure until the end of the period for academic appeals for the second examination period at the CAU. After this, my grade becomes final.

Signature: ______________________

Marking

Problem	1	2	3
Points	/36	/30	/34

Total number of points: ____________ /100

Inspection/Return

I hereby confirm that I have acknowledged the marking of this examination and that I agree with the marking noted on this cover sheet.

☐ The examination papers will remain with me. Any later objection to the marking or grading is no longer possible.

Kiel, dated ____________ Signature: ______________________

Advanced Digital Signal Processing

Examination WS 2017/2018

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Date:	09.03.2018
Time:	09:00 h – 10:30 h (90 minutes)
Location:	KS2, C-SR I

Remarks

- Please check that you have received a cover sheet plus 2 sheets with 3 problems.
- Please write your **name** and your **matriculation number** on each sheet of paper that you return.
- Please keep your student ID and your identity card ready.
- During the exam only questions concerning the problems are answered.
- Please don't use any pencil or red pen.
- Please use a **new** sheet of paper with your name and matriculation number on it for **each problem**. You can ask for more sheets of paper, if necessary.
- The exam is open books, open notes; other people are closed. Programmable electronical devices except pocket calculators are not permitted.
- Partial credit will be given. No credit will be given if an answer appears with no supporting work or reason.
- Note that the given points of the subproblems are just preliminary.
- At the end of the exam put all sheets together as you have received them, including the problem sheets.
- No one is allowed to talk or to leave his or her seat until **all** exams have been collected.
- The problems and the solutions will be published on the website of the lecture. Also the date and the place of the inspection will be announced on this website.

Problem 1 (36 points)

This problem consists of four parts (a), (b), (c) and (d). They are **not** related to each other and can be solved **independently**.

(a) Describe in a few word four different advantages of processing signals digitally compared to an analog approach. (4 P)

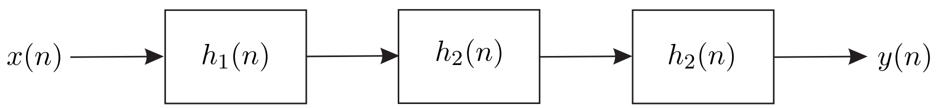
(1) Flexibility. If we want to change the continuous-time filter because of change in signal and noise characteristics, we would have to change the hardware components. Using the digital approach, we only need to modify the software. (1 P)

(2) Better control of accuracy requirements. Tolerances in continuous-time circuit components make it extremely difficult for the system designer to control the accuracy of the system. (1 P)

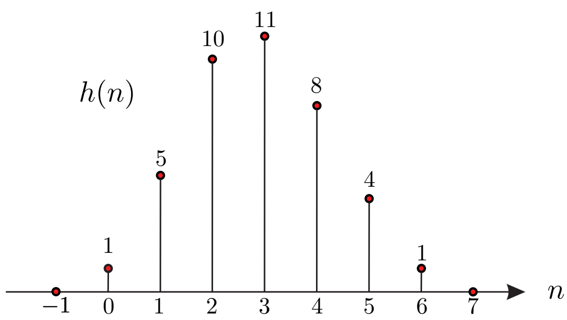
(3) The signals can be stored without further deterioration or loss of signal quality after A/D-Conversion. (1 P)

(4) Lower cost of the digital implementation. (1 P)

(b) Consider the cascade of three causal linear time-invariant systems illustrated in the following figure:



The impulse response $h_2(n)$ is given by: $h_2(n) = \gamma_{-1}(n) - \gamma_{-1}(n-2)$ and the overall impulse response is $h(n)$ as shown in the following figure



(i) Find the impulse response $h_1(n)$. **Hint:** Perform the calculation in the z-domain. (9 P)

$$h_2(n) = \gamma(n) - \gamma(n-2)$$
$$= \gamma_0(n) + \gamma_0(n-1)$$
$$H_2(z) = 1 + z^{-1}$$

Denote the overall response by H(z)

$$H(z) = H_1(z) \cdot H_2^2(z)$$
$$H_1(z) = \frac{H(z)}{H_2^2(z)}$$
$$= \frac{1 + 5z^{-1} + 10z^{-2} + 11z^{-3} + 8z^{-4} + 4z^{-5} + z^{-6}}{1 + 2z^{-1} + z^{-2}}$$

Using long division yields

$$H_1(z) = 1 + 3z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}$$

Inverse z-transform of $H_1(z)$ leads to impulse response

$$h_1(n) = \gamma_0(n) + 3\gamma_0(n-1) + 3\gamma_0(n-2) + 2\gamma_0(n-3) + \gamma_0(n-4)$$

(ii) Find the response of the overall system to the input given by (6 P)

$x(n) = \gamma_0(n) - \gamma_0(n-1)$ with $\gamma_0(n)$ being the Kronecker-Delta sequence.

Obtain $y(n)$ through calculation in z-domain:

$$x(n) = \gamma_0(n) - \gamma_0(n-1)$$
$$X(z) = 1 - z^{-1}$$
$$Y(z) = X(z) \cdot H(z)$$
$$= 1 + 4z^{-1} + 5z^{-2} + z^{-3} - 3z^{-4} - 4z^{-5} - 3z^{-6} - z^{-7}$$

Inverse z-transform leads to:

$$y(n) = \gamma_0(n) + 4\gamma_0(n-1) + 5\gamma_0(n-2) + \gamma_0(n-3) - 3\gamma_0(n-4)$$
$$- 4\gamma_0(n-5) - 3\gamma_0(n-6) - \gamma_0(n-7)$$

(c) A 3 bit A/D converter (containing a linear quantizer) has a range from -8 V to 8 V. Compute the variance of the quantization error of the inner levels when the probability distribution of the quantization error can be assumed to be uniform. (5 P)

Range of the signal: $R = \pm 8\text{V} = 16\,\text{V}$
Number of quantization levels: $L = 2^b = 2^3 = 8$ The number of levels $L = 2^3 = 8$.
The step-size $\Delta = \frac{16}{8} = 2V$
The variance of the quantization error for a uniform pdf input is given by
$\sigma_e^2 = \frac{\Delta^2}{12} = \frac{(2\text{V})^2}{12} = \frac{1}{3}\,\text{V}^2$.

(d) Given is a signal $x(n) = 2\gamma_0(n) + 2\gamma_0(n-1) + \gamma_0(n-3)$. Perform the following operations on the signal:

(i) Compute the 4-point DFT $X(\mu)$ of the signal $x(n)$ for $n = 0, 1, 2, 3$. (5 P)

$X(\mu) = \{5, 2-j, -1, 2+j\}$

(ii) Compute the 4-point inverse DFT of $Y(\mu) = X^2(\mu)$ to get $y(n)$ for $n = 0, 1, 2, 3$.
$y(n) = \{8, 8, 5, 4\}$ (5 P)

(iii) How long should the DFT length be in order to get $y(n) = x(n) * x(n)$?

$N \geq 4 + 4 - 1 = 7$ (2 P)

Problem 2 (30 points)

The problem consists of two parts (a) and (b). They are **not** related to each other and can be solved **independently**.

(a) A digital lowpass filter shall meet the following specifications: (15 P)

- Passband ripple: $\leq$ 1.2 dB
- Passband edge: 1.75 kHz
- Stopband attenuation: $\geq$ 65 dB
- Stopband edge: 2.5 kHz
- Sampling rate: 8 kHz

(i) Determine the approximate order of a (1) Butterworth and a (2) Chebyshev filter, which is required to fulfill the specifications. **Hint:** Normalized frequencies are helpful to determine the filter order. (9 P)

Calculate δ_1 and δ_2:

$$\delta_1 = 1 - 10^{-\frac{1.2}{20}} = 0.12903641$$
$$\delta_2 = 10^{-\frac{65}{20}} = 5.62341 \cdot 10^{-4}$$

δ_1 and ϵ are releated by:

$$(1 - \delta_1)^2 = \frac{1}{1 + \epsilon^2}$$
$$\epsilon^2 = \frac{1}{(1 - \delta_1)^2} - 1 = 0.3182567$$

δ is calculated with:

$$\delta = \sqrt{\frac{1}{\delta_2^2} - 1} = 1178.280157$$

Normalized frequencies for passband edge and stopband edge:

$$\Omega_{pass} = 2 \cdot \pi \cdot \frac{1.75 \cdot 10^3}{8 \cdot 10^3}$$
$$\Omega_{stop} = 2 \cdot \pi \cdot \frac{2.5 \cdot 10^3}{8 \cdot 10^3}$$

Passband and stopband edge in the analog domain:

$$\omega_{pass} = \frac{1}{T} \cdot \Omega_{pass}$$
$$\omega_{stop} = \frac{1}{T} \cdot \Omega_{stop}$$
$$\frac{\omega_{stop}}{\omega_{pass}} = \frac{10}{7}$$

Filterlengths:

$$N_{min,butter} \geq \frac{\log_{10}(\frac{1}{\delta_2^2} - 1)}{2\log_{10}(\frac{\omega_{stop}}{\omega_{pass}})} = 20.98 \rightarrow 21$$

$$N_{min,cheby} \geq \frac{\log_{10}((\sqrt{1-\delta_2^2} + \sqrt{1-\delta_2^2(1+\epsilon^2)})/\epsilon\delta_2)}{\log_{10}(\omega_{stop}/\omega_{pass} + \sqrt{(\omega_{stop}/\omega_{pass})^2 - 1})} = 8.9953 \rightarrow 9$$

(ii) Which of the two filters (Butterworth or Chebyshev) would you implement on a microcontroller with limited memory resources? Give reasons for your answer. (2 P)
With respect to the limited memory, the Chebyshev should be implemented, since it requires less filter-taps.

(iii) Sketch the typical design scheme for the frequency response of a filter (passband, stopband, transition band) and mark the above mentioned specifications (passband ripple, etc.). Additionally draw a possible Butterworth realization into your sketch. (4 P)

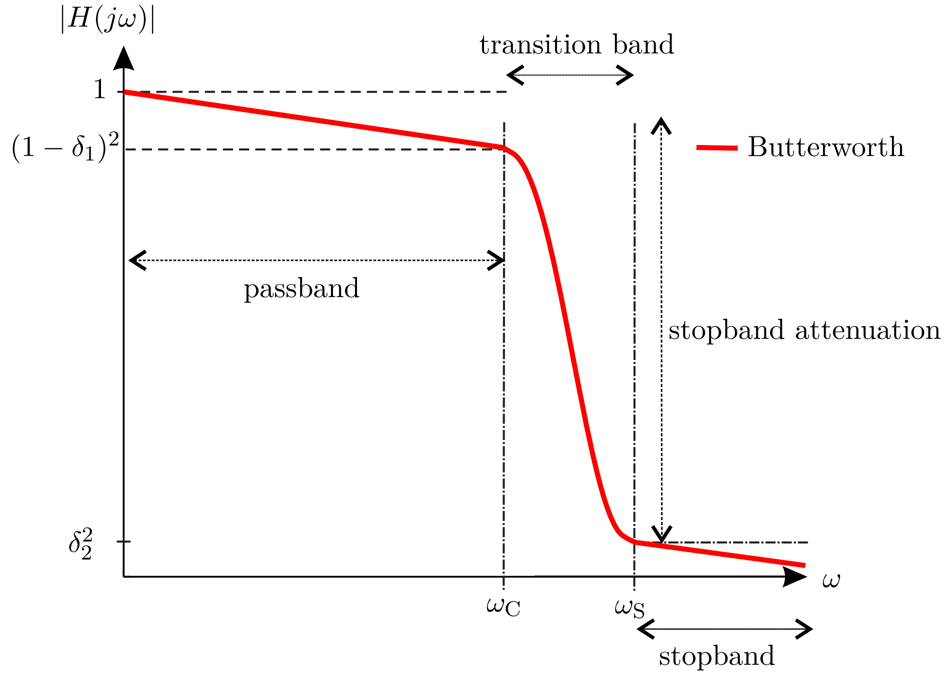


Figure 1: Design scheme of lowpass filter $|H(j\omega)|$.

(b) Given is the following transfer function: (15 P)

$$H(s) = \frac{Y(s)}{V(s)} = \frac{1}{\frac{1}{8}s^2 + \frac{3}{4}s + 1}$$

(i) Is the system with transfer function $H(s)$ stable? (2 P)

Poles:

$$\begin{aligned}\frac{1}{8}s^2 + \frac{3}{4}s + 1 &= 0\\ s^2 + 6s + 8 &= 0\\ s_{\infty 1,2} &= -3 \pm \sqrt{9-8}\\ s_{\infty_1} &= -4\\ s_{\infty_2} &= -2\end{aligned}$$

Yes, it is since all poles are located in the left s-plane (stability-criterion).

(ii) The filter $H(s)$ is used as a basis for designing a digital filter $H(z)$ by using the impulse invariance method. Determine the transfer function $H(z)$ for a sampling interval $T = 0.5$. (10 P)

Poles:

$$\begin{aligned}\frac{1}{8}s^2 + \frac{3}{4}s + 1 &= 0\\ s^2 + 6s + 8 &= 0\\ s_{\infty 1,2} &= -3 \pm \sqrt{9-8}\\ s_{\infty_1} &= -4\\ s_{\infty_2} &= -2\end{aligned}$$

Partial fraction expansion:

$$\begin{aligned}H(s) &= \frac{A_1}{s+4} + \frac{A_2}{s+2}\\ A_1 &= H(s)(s+4)|_{s=-4} = \frac{8}{s+2}|_{s=-4} = -4\\ A_2 &= H(s)(s+2)|_{s=-2} = \frac{8}{s+4}|_{s=-2} = 4\\ \Rightarrow H(s) &= \frac{-4}{s+4} + \frac{4}{s+2}\end{aligned}$$

Inverse Laplace transformation:

$$\begin{aligned}h(t) &= \left(\sum_{i=1}^{2} A_i \cdot e^{-s_{\infty_i} t}\right) \cdot \delta_{-1}(t)\\ &= 4(e^{-2t} - e^{-4t})\end{aligned}$$

Sampling:

$$\begin{aligned} h(t) &= h(n \cdot T) \\ &= 4\left(e^{-n} - e^{-2n}\right) \cdot \gamma_{-1}(n) \end{aligned}$$

z-transform:

$$\begin{aligned} H(z) &= \sum_{i=1}^{2} \frac{A_i}{1 - e^{-s_{\infty_i} T z^{-1}}} \\ &= \frac{4}{1 - e^{-1} z^{-1}} - \frac{4}{1 - e^{-2} z^{-1}} \end{aligned}$$

(iii) Sketch the signal flow graph of the resulting discrete system in direct form type I. (3 P)

Signal flow graph:

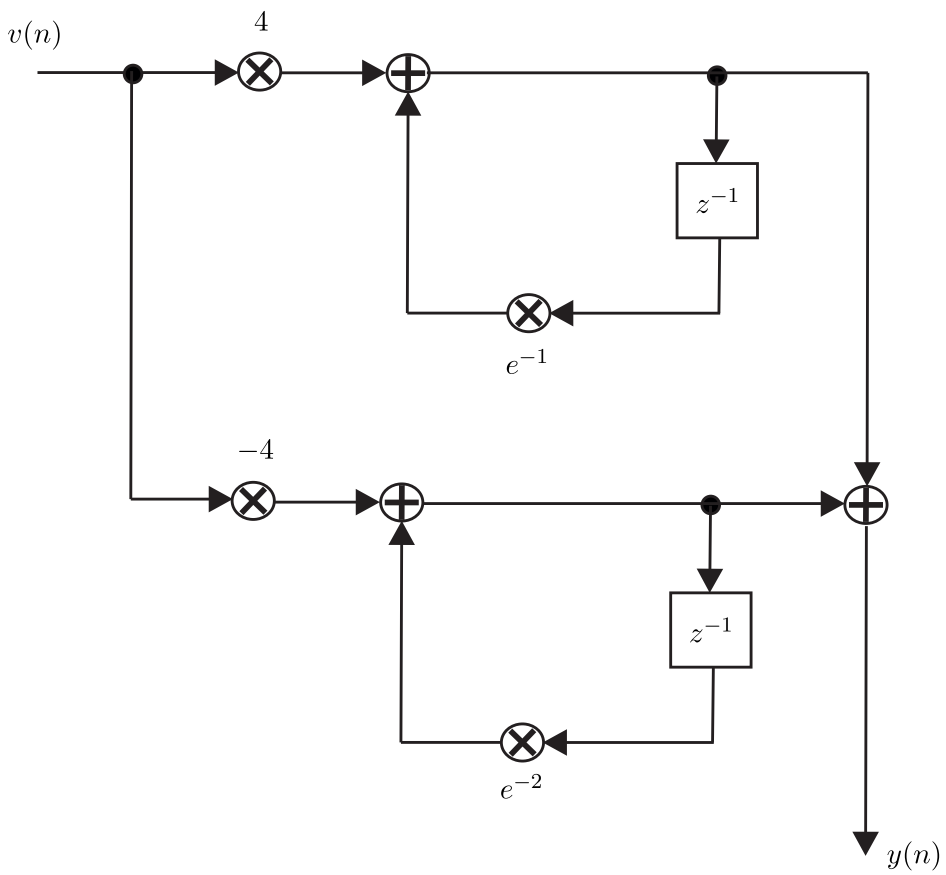
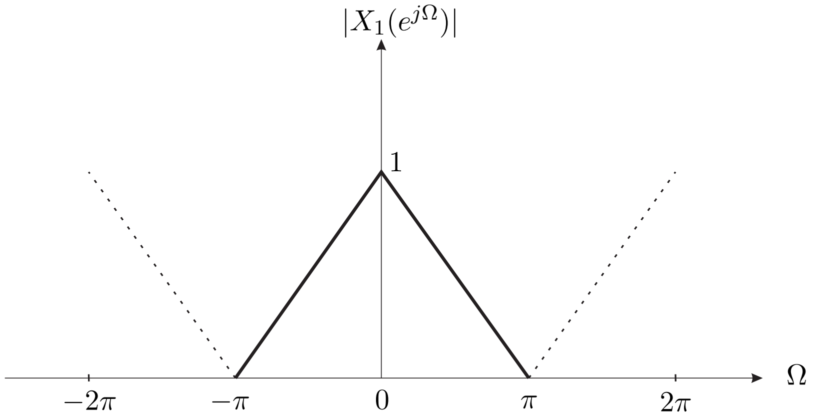


Figure 2: Signal flow graph of $H(z)$.

Problem 3 (34 points)

This problem consists of four parts (a) and (b). They are **not** related to each other and can be solved **independently**.

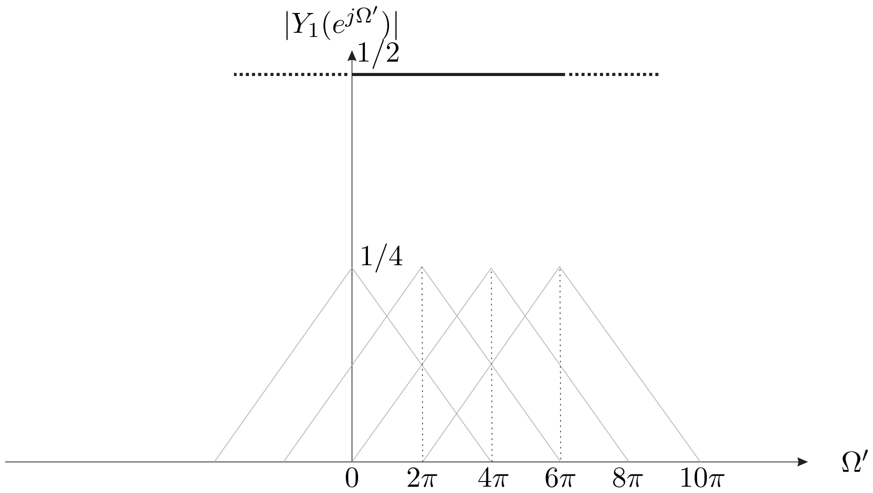
(a) A discrete signal $x_1(n)$ sampled with $T_s = 50\,\mu s$, has the spectrum $X_1(e^{j\Omega})$ illustrated in the figure below. It shall be decimated by a factor $M = 4$. (18 P)



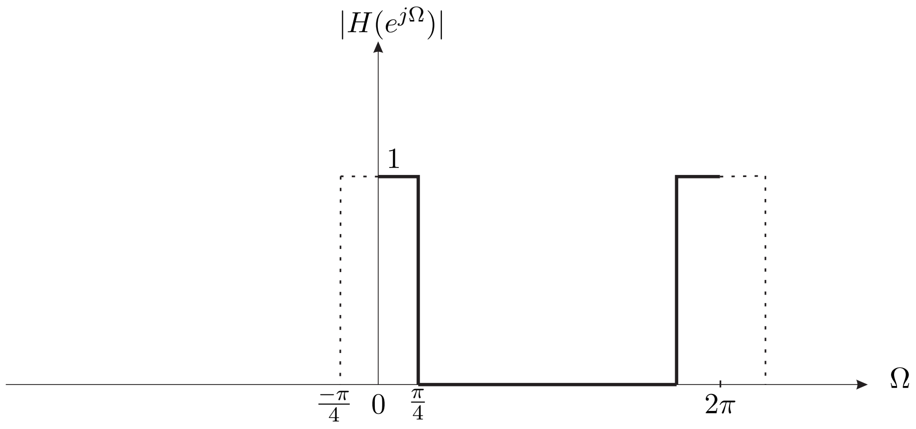
(i) Determine and sketch the output spectrum $Y_1(e^{j\Omega'}) = \mathcal{F}\{y_1(m)\}$ for $\Omega' \in [0, 10\pi]$ after the down sampler. Label all axis. (5 P)

$$Y_1(e^{j\Omega'}) = \frac{1}{4}\sum_{k=0}^{3} X_1(e^{j(\Omega'-k2\pi)/4})$$

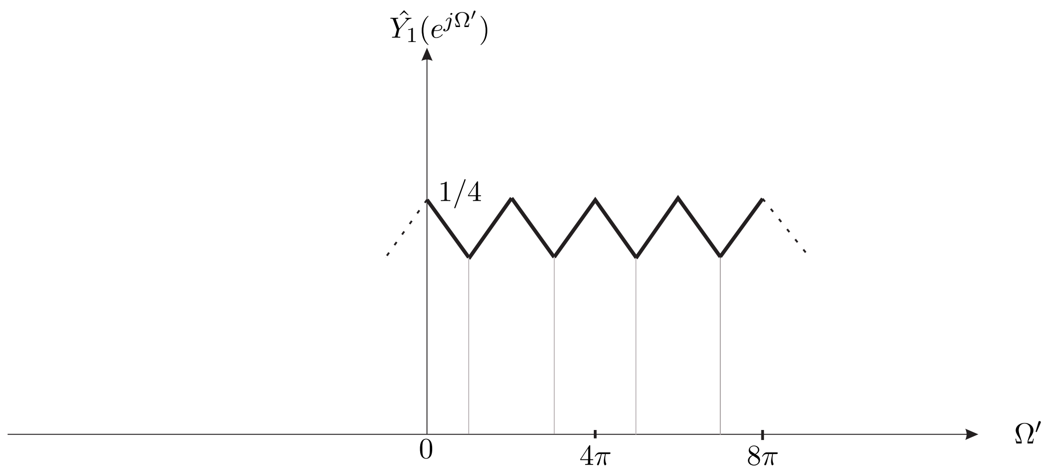
$$Y_1(e^{j\Omega'}) = \frac{1}{4}[X_1(e^{\frac{j\Omega'}{4}}) + X_1(e^{\frac{j(\Omega'-2\pi)}{4}}) + X_1(e^{\frac{j(\Omega'-4\pi)}{4}}) + X_1(e^{\frac{j(\Omega'-6\pi)}{4}})]$$



(ii) Determine the cut-off frequency for an appropriate anti-aliasing filter for $M = 4$. Please sketch the frequency response of this filter (magnitude is sufficient) in the range $\Omega \in [0, 2\pi]$. Label all axis. (3 P)

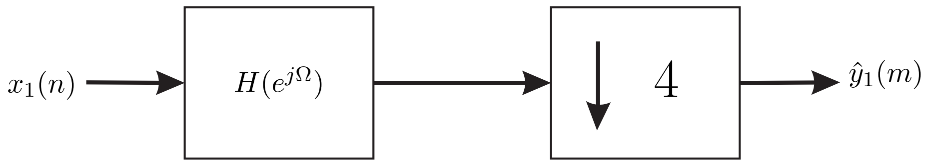


(iii) Sketch the output spectrum $\hat{Y}_1(e^{j\Omega'})$ after band limiting the input signal $X_1(e^{j\Omega})$ by the anti-aliasing filter $H(e^{j\Omega})$ from Part (ii) and after decimation by $M = 4$. Label all axis. Explain the difference to the previous output spectrum $Y_1(e^{j\Omega'})$. (3 P)

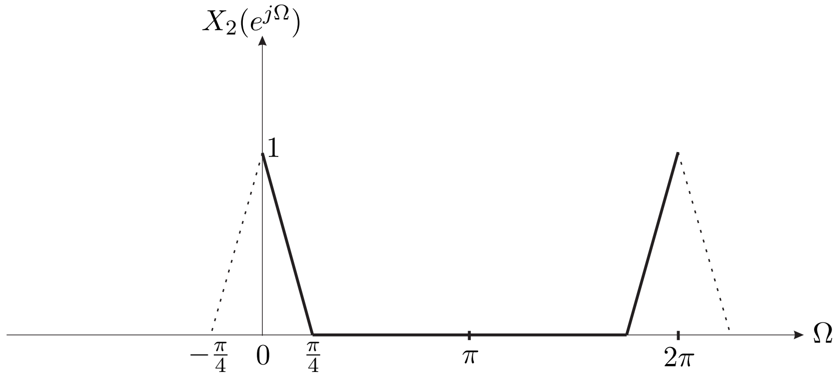


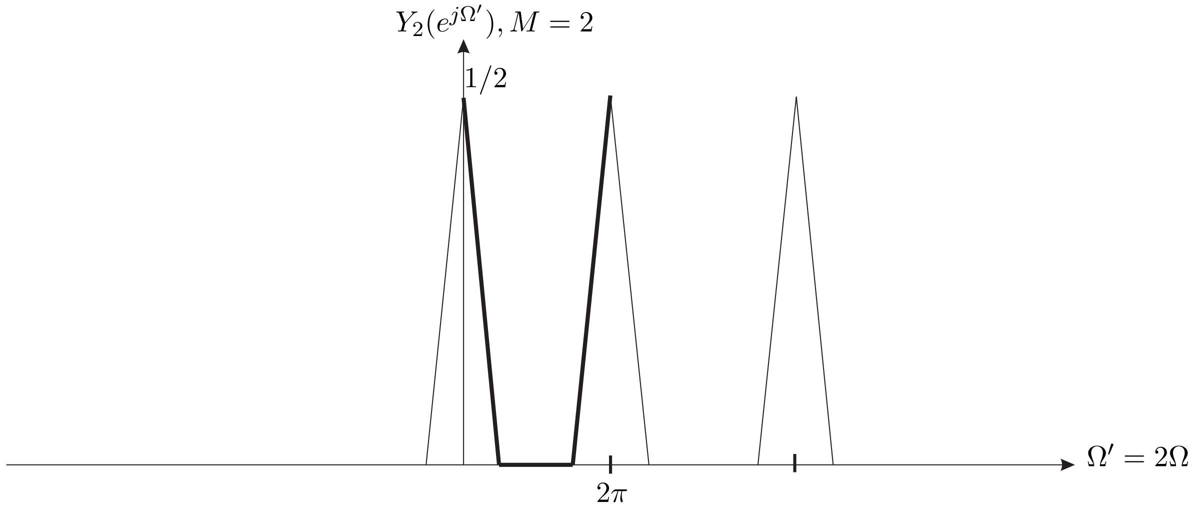
- No aliasing occurred because the input signal bandwidth has been band limited to half of the sampling rate. Nevertheless, the original signal is influenced by the band limitation!.

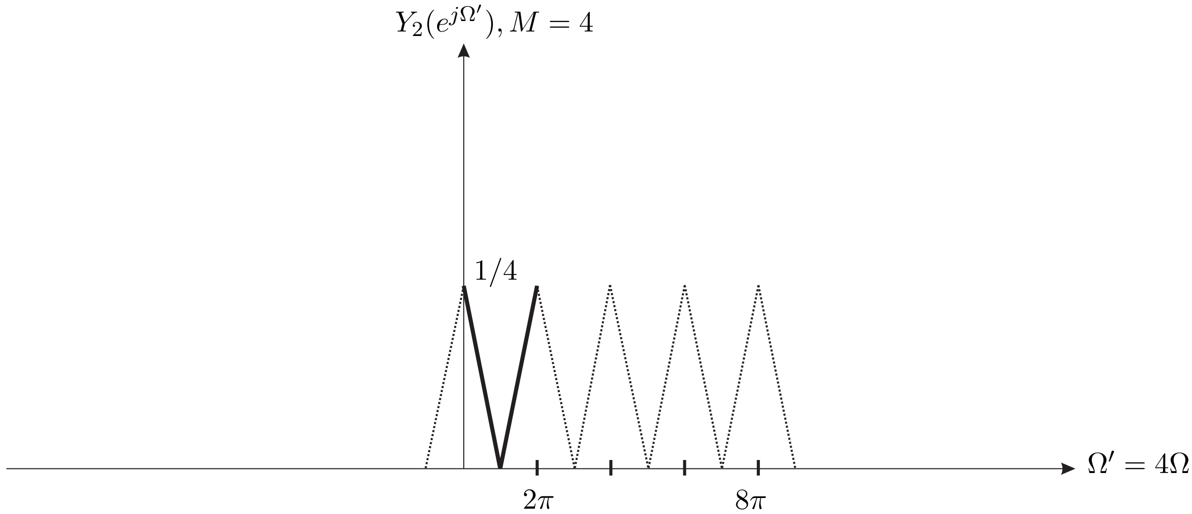
(iv) Sketch a block diagram for the complete decimator. (2 P)

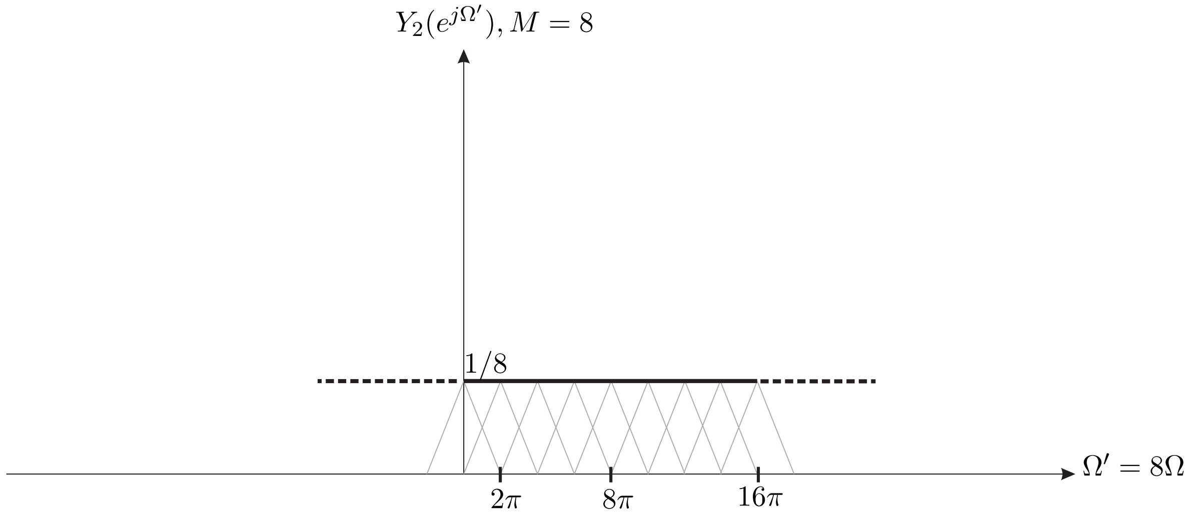


(v) Now, consider another input signal with the spectrum $X_2(e^{j\Omega})$ illustrated in the figure below. Sketch the spectrum of the down-sampled signal $Y_2(e^{j\Omega'}) = \mathcal{F}\{y_2(m)\}$ for $M = 2, 4,$ and 8 without using any anti-aliasing filter. Label all axis. For which cases does aliasing occur? (5 P)



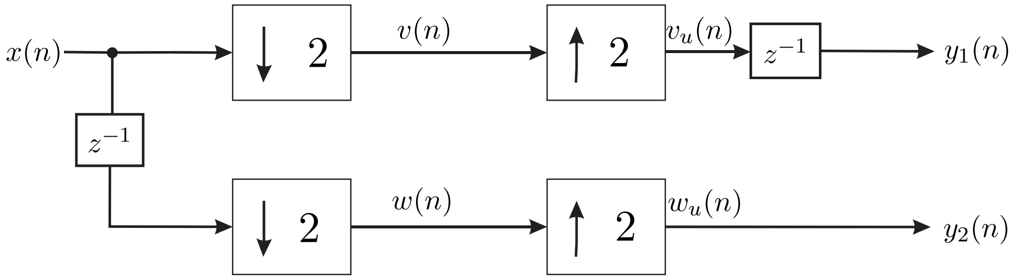






-No aliasing for $M = 2, 4$.
-Aliasing for $M = 8$.

(b) Consider the multirate system depicted in the following figure, let $x(n)$ be a real signal. (16 P)



(i) Determine $V(z), V_u(z)$, and $Y_1(z)$ as a function of $X(z)$. (6 P)

$$V(z) = \frac{1}{2}\sum_{k=0}^{1} X(z^{\frac{1}{2}} e^{-j\frac{2\pi k}{2}})$$

$$V(z) = \frac{1}{2}X(z^{\frac{1}{2}} e^{-j\frac{2\cdot\pi\cdot 0}{2}}) + \frac{1}{2}X(z^{\frac{1}{2}} e^{-j\frac{2\cdot\pi\cdot 1}{2}})$$

Substitution: $e^{-j\pi} = -1$

$$V(z) = \frac{1}{2}(X(z^{\frac{1}{2}}) + X(-z^{\frac{1}{2}}))$$

$$V_u(z) = \frac{1}{2}(X(z) + X(-z))$$

$$Y_1(z) = \frac{1}{2}(X(z) + X(-z))z^{-1}$$

(ii) Determine $W(z), W_u(z)$, and $Y_2(z)$ as a function of $X(z)$. (6 P)

$$W(z) = \frac{1}{2}\sum_{k=0}^{1} X(z^{\frac{1}{2}} e^{-j\frac{2\pi k}{2}})z^{-\frac{1}{2}}$$

$$W(z) = \frac{1}{2}X(z^{\frac{1}{2}} e^{-j\frac{2\cdot\pi\cdot 0}{2}})z^{-\frac{1}{2}} + \frac{1}{2}X(z^{\frac{1}{2}} e^{-j\frac{2\cdot\pi\cdot 1}{2}})z^{-\frac{1}{2}}$$

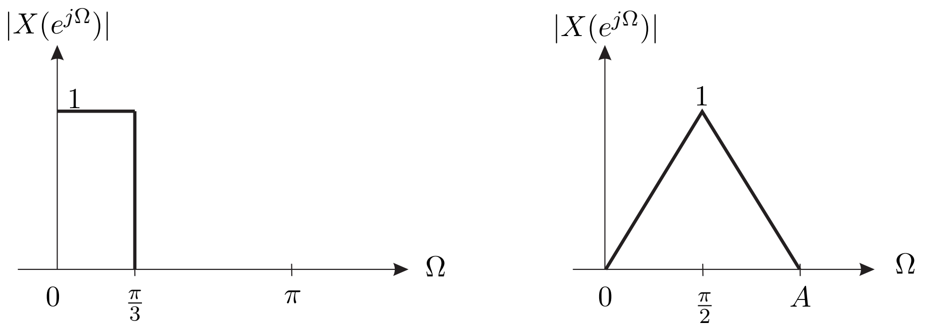
Substitution: $e^{-j\pi} = -1$

$$W(z) = \frac{1}{2}\Big(X(z^{\frac{1}{2}})z^{-\frac{1}{2}} + z^{-\frac{1}{2}}X(-z^{\frac{1}{2}})\Big)$$

$$Y_2(z) = W_u(z) = \frac{1}{2}\Big(X(z)z^{-1} + z^{-1}X(-z)\Big)$$

$$= \frac{1}{2}\Big(X(z) + X(-z)\Big)z^{-1}$$

(iii) Consider the two possible spectra of the input signal $x(n)$ shown in figure A and B, in which case does aliasing occur with the given multirate system? Give reasons to your answer. (4 P)



A *B*

In case A, no aliasing is occurring while in case B aliasing will occur.