Advanced Digital Signal Processing

Examination WS 2014/2015

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Date:	06.03.2015
Name:	______________________________
Matriculation Number:	______________________________

Declaration of the candidate before the start of the examination

I hereby confirm that I am registered for, authorised to sit and eligible to take this examination.
I understand that the date for inspecting the examination will be announced by the EE&IT Examination Office, as soon as my provisional examination result has been published in the QIS portal. After the inspection date, I am able to request my final grade in the QIS portal. I am able to appeal against this examination procedure until the end of the period for academic appeals for the second examination period at the CAU. After this, my grade becomes final.

Signature: ______________________________

Marking

Problem	1	2	3
Points	/30	/40	/30

Total number of points: ________________ /100

Inspection/Return

I hereby confirm that I have acknowledged the marking of this examination and that I agree with the marking noted on this cover sheet.

☐ The examination papers will remain with me. Any later objection to the marking or grading is no longer possible.

Kiel, dated ________________ Signature: ______________________________

Advanced Digital Signal Processing

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Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Date:	06.03.2015
Time:	09:00 h – 10:30 h (90 minutes)
Location:	C-SR I

Remarks

- Please check that you have received a cover sheet plus 5 sheets with 3 problems.
- Please write your **name** and your **matriculation number** on each sheet of paper that you return.
- Please keep your student ID and your identity card ready.
- During the exam only questions concerning the problems are answered.
- Please don't use any pencil or red pen.
- Please use a **new** sheet of paper with your name and matriculation number on it for **each problem**. You can ask for more sheets of paper, if necessary.
- The exam is open books, open notes; other people are closed. Programmable electronical devices except pocket calculators are not permitted.
- Partial credit will be given. No credit will be given if an answer appears with no supporting work or reason.
- Note that the given points of the subproblems are just preliminary.
- At the end of the exam put all sheets together as you have received them, including the problem sheets.
- No one is allowed to talk or to leave his or her seat until **all** exams have been collected.
- The problems and the solutions will be published on the website of the lecture. Also the date and the place of the inspection will be announced on this website.

Problem 1 (30 points)

This question consists of three parts (a), (b), and (c). Each part can be solved independently.

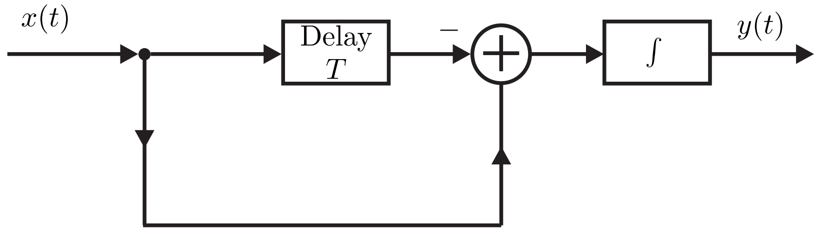

Figure 1: Hold circuit realization with a summer and an integrator.

(a) One realization of a practical zero-order hold circuit is presented in Fig. 1. (10 P)

(i) Find the unit impulse response of this circuit. Sketch it.

(ii) Find and sketch the magnitude frequency response of the above impulse response.

(b) Given is a signal $f(n)$ that is time limited to 10 ms and has an essential bandwidth of 10 kHz.

(i) For the case where the number of signal points is equal to the number of FFT points ($L = M$) determine L the number of signal samples necessary to compute a radix-2 FFT with a frequency resolution of at least 50 Hz. (4 P)

(ii) What is the maximum sampling interval at which the signal must be sampled in order to retain the information present within the 10 kHz bandwidth. (2 P)

(iii) Does the signal need to be zero padded? If yes, for how much duration (in ms)? Explain your answer. (4 P)

(c) Using the Fourier transform find the response $y(n)$ of the causal system with the frequency response (10 P)

$$H(e^{j\Omega}) = \frac{e^{j\Omega} + 0.32}{e^{j2\Omega} + e^{j\Omega} + 0.16}$$

to the input

$$x(n) = (-0.5)^n \delta_{-1}(n),$$

with

$$\delta_{-1}(n) = \begin{cases} 1, \text{for} \quad n \geq 1, \\ 0, \text{else}. \end{cases}$$

Problem 2 (40 points)

This question consists of three parts (a), (b), and (c). Each part can be solved independently. Do each rounding to 2 decimals.

(a) An analog Butterworth filter has the following transfer function:

$$H(s) = \frac{6.25}{s^2 + 4s + 6.25}.$$

The edge frequencies in radiants/s are $\omega_\mathrm{P} = 1$ for the passband and $\omega_\mathrm{S} = 5$ for the stopband.

(i) Determine the passband and stopband ripple of $|H(j\omega)|$ and sketch the tolerance scheme. (5 P)

(ii) The filter $H(s)$ is used as a basis for designing a digital filter $H_\mathrm{I}(z)$ by using the impulse invariance method.

Determine the transfer function $H_\mathrm{I}(z)$ and the corresponding impulse response $h_\mathrm{I}(n)$ for a sampling interval $T = 0.1$.

Rewrite the impulse response $h_\mathrm{I}(n)$ such, that it doesn't contain any imaginary unit j. (12 P)

(b) A real valued discrete system $G(z)$ is causal and non-recursive. The system meets the following characteristics:

1. Constant signals are suppressed.
2. Signals with half of the sampling frequency are suppressed as well.
3. The system has the minimum number of zeros, that are necessary to fulfill the characteristics 1 and 2.
4. The system is a bandpass with a maximum system gain of 4 at $\Omega = \frac{\pi}{2}$.

(i) Determine the transfer function $G(z)$. (8 P)

(ii) Give the difference equation of the system for an input signal for an input signal $v(n)$ and the output $y(n)$. (2 P)

(c) An ideal Hilbert transformer has the following impulse response:

$$h_\mathrm{H}(n) = \frac{1 - \cos(\pi n)}{\pi n}.$$

For practical purposes the impulse response of the hilbert transformer will be shortened to

$$h_\mathrm{S,H}(n) = \begin{cases} h_\mathrm{H}(n), & \text{for } |n| \leq 5, \\ 0, & \text{else} \end{cases}.$$

(i) Determine the frequency response $H_{\mathrm{S,H}}(e^{j\Omega})$. (5 P)

(ii) The shortened impulse response $h_{\mathrm{S,H}}(n)$ is not causal. Determine the minimum shift in samples to obtain a causal impulse response $h_{\mathrm{C}}(n)$. Determine $h_{\mathrm{C}}(n)$ in dependency of $h_{\mathrm{S,H}}(n)$. (1 P)

(iii) Determine the corresponding transfer function $H_{\mathrm{C}}(e^{j\Omega})$ in dependency of $H_{\mathrm{S,H}}(e^{j\Omega})$. (2 P)

(iv) Determine the phase $\phi(\Omega)$ of $H_{\mathrm{C}}(e^{j\Omega})$ for $|\omega| \leq \pi$. (3 P)

(v) Is the system a linear phase filter. Explain your answer. (2 P)

Problem 3 (30 points)

Parts (a), (b), and (c) can be solved independently.

(a) Consider the analysis-synthesis system shown in Fig. 2. The lowpass filter $h_0(n)$ is identical in the analysis and synthesis bank, and the highpass filter $h_1(n)$ is identical in the analysis and synthesis bank. The Fourier transforms of $h_0(n)$ and $h_1(n)$ are related by

$$H_1(e^{j\Omega}) = H_0(e^{j(\Omega+\pi)})$$

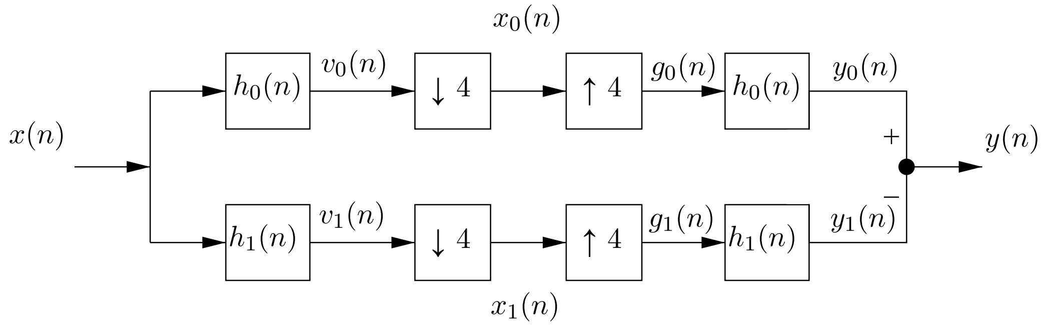

Figure 2: System

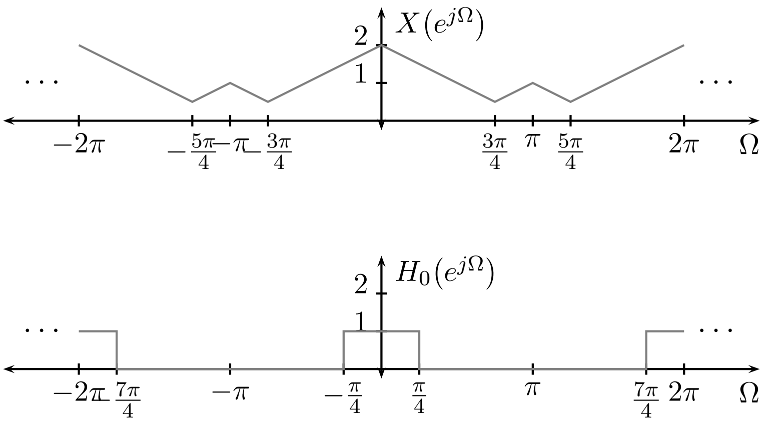

Figure 3: Fourier spectra

(i) If $X(e^{j\Omega})$ and $H_0(e^{j\Omega})$ are shown as in Fig. 3, sketch $V_0(e^{j\Omega})$, $X_0(e^{j\Omega})$, $G_0(e^{j\Omega})$, and $Y_0(e^{j\Omega})$ in the range $|\Omega| \leq 4\pi$. (8 P)

(ii) Write a general expression for $G_0(e^{j\Omega})$ in terms of $X(e^{j\Omega})$ and $H_0(e^{j\Omega})$. (**Note:** Do not assume that $X(e^{j\Omega})$ and $H_0(e^{j\Omega})$ are as shown in Fig. 3). (6 P)

(iii) Give a general expression for the output Fourier spectrum $Y(e^{j\Omega})$ in terms of $X(e^{j\Omega})$ and $H_0(e^{j\Omega})$. (6 P)

(iv) Sketch the Fourier spectrum $Y(e^{j\Omega})$ of the output signal $y(n)$ in the range $|\Omega| \leq 4\pi$, if now again $H_0(e^{j\Omega})$ and $X(e^{j\Omega})$ have the shape as given in Fig. 3. (4 P)

(b) Consider a signal processing tool which is only able to process audio signals at a sampling rate of 8 kHz. In a database you find an audio signal with a sampling rate of 10 kHz. You want to use this signal to test a new algorithm you implemented in the tool. Sketch a block diagram of a system which is capable of performing the required sample rate conversion in order to be able to use the signal. If you use any filters, please plot the frequency response (magnitude is sufficient) of those filters. (5 P)

(c) Given the structure in Fig. 4, give a general criterion for the filter $H_0(e^{j\Omega})$ such that no aliasing occurs. (1 P)

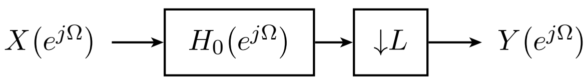

Figure 4: Block diagram of a decimation