Advanced Digital Signal Processing

Examination SS 2018

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Date:	20.09.2018
Name:	______________________________
Matriculation Number:	______________________________

Declaration of the candidate before the start of the examination

I hereby confirm that I am registered for, authorised to sit and eligible to take this examination.
I understand that the date for inspecting the examination will be announced by the EE&IT Examination Office, as soon as my provisional examination result has been published in the QIS portal. After the inspection date, I am able to request my final grade in the QIS portal. I am able to appeal against this examination procedure until the end of the period for academic appeals for the second examination period at the CAU. After this, my grade becomes final.

Signature: ________________________________

Marking

Problem	1	2	3
Points	/33	/38	/29

Total number of points: _________________ /100

Inspection/Return

I hereby confirm that I have acknowledged the marking of this examination and that I agree with the marking noted on this cover sheet.

☐ The examination papers will remain with me. Any later objection to the marking or grading is no longer possible.

Kiel, dated _________________ Signature: ________________________________

Advanced Digital Signal Processing

Examination SS 2018

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Date:	20.09.2018
Time:	09:00 h – 10:30 h (90 minutes)
Location:	KS2, C-SR I

Remarks

- Please write your **name** and your **matriculation number** on each sheet of paper that you return.
- Please keep your student ID and your identity card ready.
- You may not start working on the exam until you are specifically told to do so.
- During the exam only questions concerning the problems are answered.
- Please use a **new** sheet of paper with your name and matriculation number on it for **each problem**. You can ask for more sheets of paper, if necessary.
- Please don't use any pencil or red pen.
- The exam is open books, open notes; other people are closed. All electronic devices except non-programmable pocket calculators are prohibited.
- Partial credit will be given. No credit will be given if an answer appears with no supporting work or reason. All axes in sketches must labeled to receive full credit.
- Note that the given points of the subproblems are just preliminary.
- You will be informed about the approaching end of the exam. This will be done orally five and one minute prior to the end of the exam. Once the end of the exam has been announced, you must **stop writing immediately**.
- In case you should feel negatively impacted by your surroundings during the exam, you must notify the exam supervisor immediately.
- At the end of the exam put all sheets together as you have received them, including the problem sheets and the **sheet with your signature** on it.
- No one is allowed to talk or to leave his or her seat until **all** exams have been collected. Any talking during this time may be considered an **attempt of deception**.
- The problems and the solutions will be published on the website of the lecture. Also the date and the place of the inspection will be announced on this website.

Problem 1: (33 points)

*This problem consists of four parts (a), (b) and (c). They are **not** related to each other and can be solved **independently**.*

(a) Given are two time-continuous signals (9 P)

$$x(t) = 10 + 8 \cdot \sin\left(\frac{2000\pi}{\text{s}} t\right) - 5 \cdot \cos\left(\frac{4000\pi}{\text{s}} t\right)$$
$$y(t) = 10 + 8 \cdot \cos\left(\frac{2000\pi}{\text{s}} t\right)$$

(i) Determine the Fourier transform $X(j\omega)$ and $Y(j\omega)$ for the signals $x(t)$ and $y(t)$ respectively. (4 P)

$$X(j\omega) = 2\pi\Big(10\delta_0(\omega) + \frac{4}{j}(\delta_0(\omega - \frac{2000\pi}{\text{s}}) - \delta_0(\omega + \frac{2000\pi}{\text{s}})) - \frac{5}{2}(\delta_0(\omega - \frac{4000\pi}{\text{s}}) + \delta_0(\omega + \frac{4000\pi}{\text{s}}))\Big)$$

$$Y(j\omega) = 2\pi\Big(10\delta_0(\omega) + 4(\delta_0(\omega - \frac{2000\pi}{\text{s}}) + \delta_0(\omega + \frac{2000\pi}{\text{s}}))\Big)$$

(ii) The signals should be processed by a time-discrete system. Name the fundamental elements of a digital processing system which are required for digitalization. (3 P)

(1) Anti-aliasing filter (Lowpass)

(2) Sample and Hold (S/H) Circuit

(3) A/D-Converter

(iii) What is the minimum sampling frequency f_s accordingly to the Nyquist theorem that has to be used? (2 P)

$$f_\text{max} = 2 \text{ kHz} => f_\text{s} = 4 \text{ kHz}$$

(b) Now assume that both time-continuous signals have been sampled with a sampling frequency of $f_\text{s} = 12$ kHz. (10 P)

(i) Determine the time-discrete signals $x(n)$ and $y(n)$. (4 P)

$$t = nT_\text{s} = \frac{n}{f_\text{s}}$$

$$x(n) = 10 + 8 \cdot \sin\left(\frac{\pi}{6} n\right) - 5 \cdot \cos\left(\frac{\pi}{3} n\right)$$
$$y(n) = 10 + 8 \cdot \cos\left(\frac{\pi}{6} n\right)$$

(ii) The signal $y(n)$ is a result of $x(n)$ convolved with a system $h(n)$. What kind of a filter is the system h (lowpass, bandpass or highpass)? Give short reasons to your answer. (3 P)

Lowpass filter because all frequencies till $\Omega = \pi/6$ are passed through. $\Omega = \pi/3$ is not passed through.

(iii) Give values of the transfer function of $H(e^{j\Omega})$ at $\Omega = 0, \pi/6$ and $\pi/3$. (3 P)

$$H(e^{j0}) = 1$$
$$H(e^{j\frac{\pi}{6}}) = e^{j\frac{\pi}{2}} = j$$
$$H(e^{j\frac{\pi}{3}}) = 0$$

(c) Now consider an input signal $x(n) = \{1, 5, -2, -1, 6, 1, -2, 5, 1\}$ with $x(n) = 0$ for $n < 0, n > 8$ into a system $h(n) = \{2, 4, 2\}$ (14 P)

(i) Compute the corresponding output $y(n)$ of the system. (7 P)

$$y(n) = \{2, 14, 18, 0, 4, 24, 12, 4, 18, 14, 2\}$$

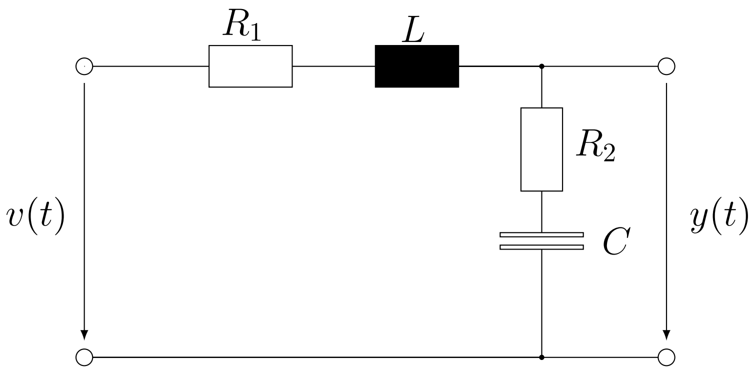
(ii) Compute the 8-point DFT $H(\mu)$ of $h(n)$. (7 P)

$$H(\mu) = \sum_{n=0}^{7} h(n) \cdot W_8^{\mu n} = 2 + 4W_8^{\mu} + 2W_8^{2\mu}$$
$$H(0) = 8$$
$$H(1) = 2 + 2\sqrt{2} - j(2 + 2\sqrt{2})$$
$$H(2) = -4j$$
$$H(3) = 2 - 2\sqrt{2} + j(2 - 2\sqrt{2})$$
$$H(4) = 0$$
$$H(5) = H^*(3) = 2 - 2\sqrt{2} - j(2 - 2\sqrt{2})$$
$$H(6) = H^*(2) = 4j$$
$$H(7) = H^*(1) = 2 + 2\sqrt{2} + j(2 + 2\sqrt{2})$$

Problem 2: (38 points)

The problem consists of two parts (a) and (b). They are **not** related to each other and can be solved **independently**.

(a) Designing aliasing-filter for AD-converters is crucial to obtain correct values. Given is the following network: (17 P)



Values: $R_1 = 100\,\text{k}\Omega$, $R_2 = 100\,\Omega$, $C = 10\,\text{mF}$, $L = 3\,\text{mH}$

(i) Determine the transfer function of the filter as a function of R_1, R_2, L and C: $H(s) = \frac{Y(s)}{V(s)}$ (6 P)

Hint: If you could not solve (i), assume $H(s) = \frac{\frac{1}{sC}+R_1}{R_1+sL+\frac{1}{sC}+R_2}$

Voltage divider: $H(s) = \frac{\frac{1}{sC}+R_2}{R_1+sL+\frac{1}{sC}+R_2} = \frac{\frac{1}{C}+sR_2}{s^2L+sR_1+sR_2+\frac{1}{C}}$

(ii) Calculate the poles and zeroes of the transfer function. Is the filter stable? (3 P)

Poles:

$$s^2L + sR_1 + sR_2 + \frac{1}{C} = 0$$

$$s_{\infty_{1,2}} = -\frac{R_1+R_2}{2L} \pm \sqrt{\left(\frac{R_1+R_2}{2L}\right)^2 - \frac{1}{LC}}$$

$$s_{\infty_1} = -9,99 \cdot 10^{-4}$$

$$s_{\infty_2} = -333,6 \cdot 10^5$$

Zeros:

$$\frac{1}{sC} + R_2 = 0$$

$$1 + sCR_2 = 0$$

$$s_{0_1} = -\frac{1}{CR_2}$$

$$s_{0_2} = -1$$

(iii) The samplerate of the A/D-converter is set to $f_s = 44100\,Hz$. For a good alias rejection at least 50 dB attenuation is needed. What is the necessary cut-off frequency for the filter and how much attenuation does it provide at this frequency? Does this filter meet the requirements? (5 P)

$H(s) = \frac{\frac{1}{sC}+R_2}{R_1+sL+\frac{1}{sC}+R_2}$
Use $\frac{f_s}{2}$ as the evaluated frequency: $\frac{f_s}{2} = 44100\pi$
$|H(s = j44100\pi)| = |\frac{100-j7.22}{100100+j415.63}| \approx 1 \cdot 10^{-3} \approx -60\text{dB}$
Yes, the requirements are met.

(iv) Is it possible to use a digital aliasing filter after the A/D-converter instead of an analogue one? Give reasons to your answer. (3 P)

No! Aliasing has already occured in the A/D-Converter during conversion in that case.

(b) A filter with the following transfer function is given: (13 P)

$$H(s) = \frac{8}{s^2 + 8s + 7}$$

(i) Is the system $H(s)$ stable? (2 P)
Poles:

$$\begin{aligned} s^2 + 8s + 7 &= 0 \\ s_{\infty 1,2} &= -4 \pm \sqrt{16-7} \\ s_{\infty 1} &= -7 \\ s_{\infty 2} &= -1 \end{aligned}$$

System is stable.

(ii) The filter $H(s)$ is used as a basis for designing a digital filter $H(z)$ by using the impulse invariance method. Determine the transfer function $H(z)$ for a sampling interval $T = 0.2$. (11 P)
Poles:

$$\begin{aligned} s^2 + 8s + 7 &= 0 \\ s_{\infty 1,2} &= -4 \pm \sqrt{16-7} \\ s_{\infty 1} &= -7 \\ s_{\infty 2} &= -1 \end{aligned}$$

Partial fraction expansion:

$$H(s) = \frac{A_1}{s+7} + \frac{A_2}{s+1}$$
$$A_1 = H(s)(s+7)|_{s=-7} = \frac{8}{s+1}|_{s=-7} = -\frac{4}{3}$$
$$A_2 = H(s)(s+1)|_{s=-1} = \frac{8}{s+7}|_{s=-1} = \frac{4}{3}$$
$$\Rightarrow H(s) = \frac{-\frac{4}{3}}{s+7} + \frac{\frac{4}{3}}{s+1}$$

Inverse Laplace transformation:

$$h(t) = \left(\sum_{i=1}^{2} A_i \cdot e^{-s_{\infty_i} t}\right) \cdot \delta_{-1}(t)$$
$$= \frac{4}{3}(e^{-t} - e^{-7t}) \cdot \delta_{-1}(t)$$

Sampling:

$$h(t) = h(n \cdot T)$$
$$= \frac{4}{3}(e^{-0.2n} - e^{-1.4n}) \cdot \gamma_{-1}(n)$$

z-transform:

$$H(z) = \sum_{i=1}^{2} \frac{A_i}{1 - e^{-s_{\infty_i} T} z^{-1}}$$
$$= \frac{\frac{4}{3}}{1 - e^{-0.2} z^{-1}} - \frac{\frac{4}{3}}{1 - e^{-1.4} z^{-1}}$$

(c) A filter with the following transfer function is given: (8 P)

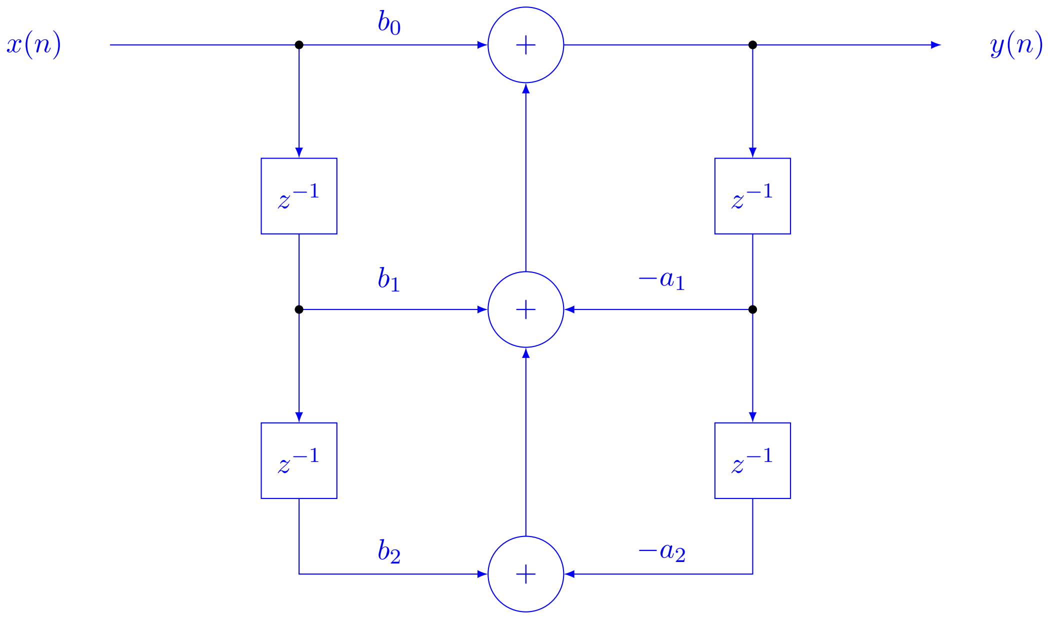
$$H(z) = \frac{(z+1)^2}{(z - \frac{3}{5})(z + \frac{2}{3})}$$

(i) Determine the coefficients for the signal flow graph a_1, a_2, b_0, b_1 and b_2. (2 P)

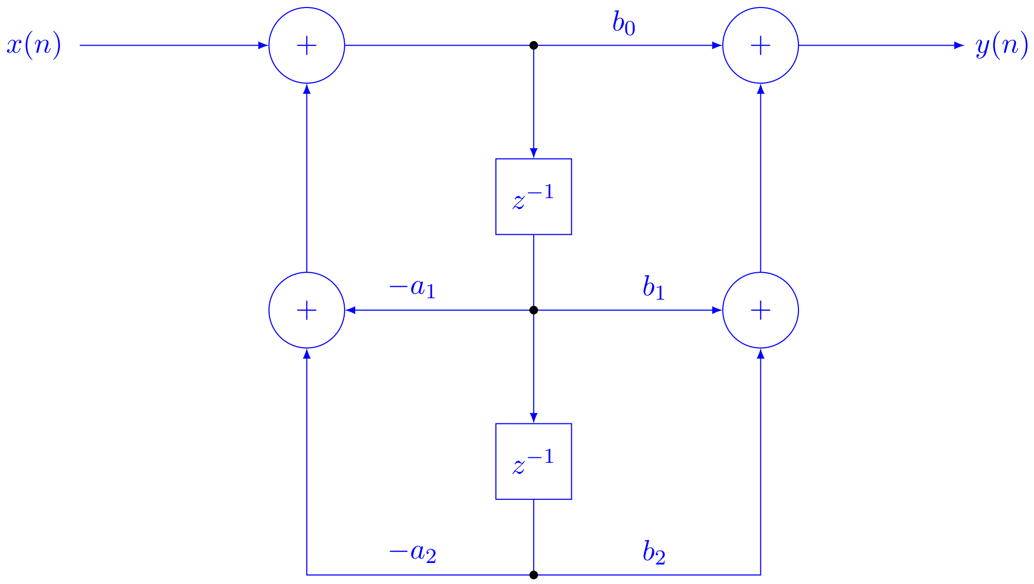
$$H(z) = \frac{(z+1)^2}{(z - \frac{3}{5})(z + \frac{2}{3})} \qquad = \frac{z^2 + 2z + 1}{z^2 + \frac{1}{15} z - \frac{2}{5}} = \frac{1 + 2z^{-1} + z^{-2}}{1 + \frac{1}{15} z^{-1} - \frac{2}{5} z^{-2}}$$

$a_1 = \frac{1}{15}, a_2 = -\frac{2}{5}, b_0 = 1, b_1 = 2, b_2 = 1$

(ii) Sketch the signal flow graph in direct form I. (3 P)

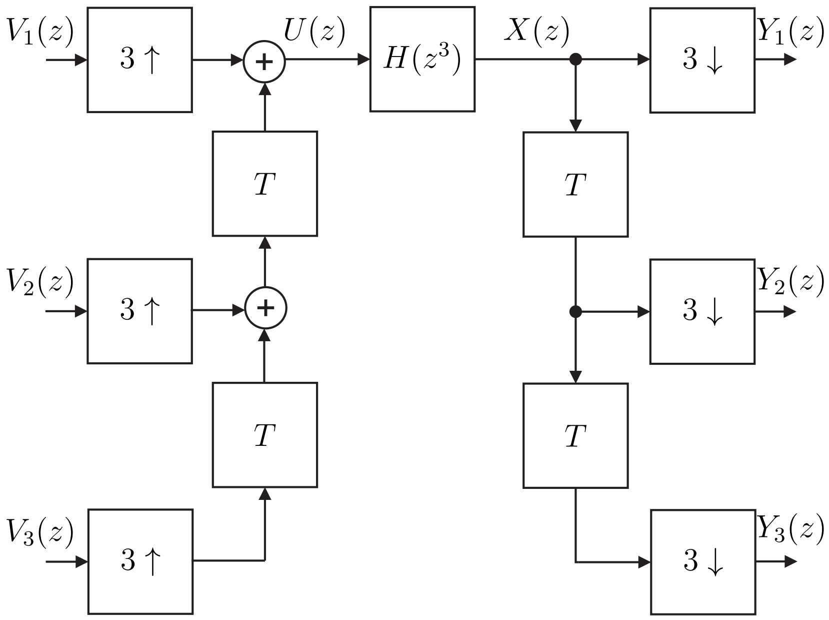


(iii) Sketch the signal flow graph in direct form II. (3 P)



Problem 3: (29 points)

Given is the following multirate system where $H(z^3)$ is a digital filter.



(a) How are $U(z)$ and $X(z)$ related to the input signals $V_i(z)$ with $i \in [1, 2, 3]$. (4 P)

$$U(z) = V_1(z^3) + V_2(z^3)z^{-1} + V_3(z^3)z^{-2}$$
$$X(z) = U(z)H(z^3) = \Big(V_1(z^3) + V_2(z^3)z^{-1} + V_3(z^3)z^{-2}\Big)H(z^3)$$

(b) Give also the expressions for the output signals $Y_i(z)$ with $i \in [1, 2, 3]$ using $U(z)$ and $H(z^3)$. (6 P)

$$Y_1(z) = \frac{1}{3}\sum_{k=0}^{2} U(z^{1/3}W_3^k)H((z^{1/3}W_3^k)^3) = \frac{1}{3}\sum_{k=0}^{2} U(z^{1/3}W_3^k)H(z)$$
$$Y_2(z) = \frac{1}{3}\sum_{k=0}^{2} U(z^{1/3}W_3^k)H((z^{1/3}W_3^k)^3)(z^{1/3}W_3^k)^{-1} = \frac{1}{3}\sum_{k=0}^{2} U(z^{1/3}W_3^k)H(z)(z^{1/3}W_3^k)^{-1}$$
$$Y_3(z) = \frac{1}{3}\sum_{k=0}^{2} U(z^{1/3}W_3^k)H((z^{1/3}W_3^k)^3)(z^{1/3}W_3^k)^{-2} = \frac{1}{3}\sum_{k=0}^{2} U(z^{1/3}W_3^k)H(z)(z^{1/3}W_3^k)^{-2}$$

(c) Determine the outputs $Y_i(z)$ in dependency of $V_i(z)$ with $i \in [1, 2, 3]$. (10 P)

$$
\begin{aligned}
Y_1(z) =& \frac{1}{3}\sum_{k=0}^{2}\Big(V_1((z^{1/3}W_3^k)^3) \\
& + V_2((z^{1/3}W_3^k)^3)(z^{1/3}W_3^k)^{-1} \\
& + V_3((z^{1/3}W_3^k)^3)(z^{1/3}W_3^k)^{-2}\Big)H(z) \\
=& H(z)\frac{1}{3}\left(3V_1(z) + z^{-1/3}3V_2(z)\underbrace{\sum_{k=0}^{2}W_3^{-k}}_{=0} + z^{-2/3}3V_3(z)\underbrace{\sum_{k=0}^{2}W_3^{-2k}}_{=0}\right) \\
=& V_1(z)H(z)
\end{aligned}
$$

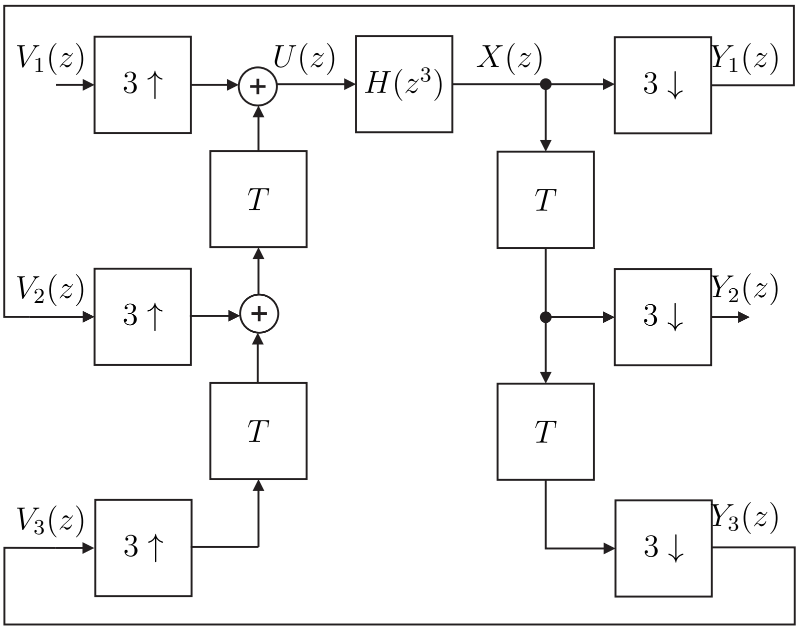
$$
\begin{aligned}
Y_2(z) =& \frac{1}{3}\sum_{k=0}^{2}\Big(V_1((z^{1/3}W_3^k)^3)(z^{1/3}W_3^k)^{-1} \\
& + V_2((z^{1/3}W_3^k)^3)(z^{1/3}W_3^k)^{-2} \\
& + V_3((z^{1/3}W_3^k)^3)(z^{1/3}W_3^k)^{-3}\Big)H(z) \\
=& V_3(z)H(z)z^{-1}
\end{aligned}
$$

$$
\begin{aligned}
Y_3(z) =& \frac{1}{3}\sum_{k=0}^{2}\Big(V_1((z^{1/3}W_3^k)^3)(z^{1/3}W_3^k)^{-2} \\
& + V_2((z^{1/3}W_3^k)^3)(z^{1/3}W_3^k)^{-3} \\
& + V_3((z^{1/3}W_3^k)^3)(z^{1/3}W_3^k)^{-4}\Big)H(z) \\
=& V_2(z)H(z)z^{-1}
\end{aligned}
$$

Assume now that the output $Y_3(z)$ is connected to $V_3(z)$ and $Y_1(z)$ is connected to $V_2(z)$

$$
\begin{aligned}
V_3(z) &= Y_3(z) \\
V_2(z) &= Y_1(z)
\end{aligned}
$$

(d) Sketch the modified System with all signals. (4 P)



(e) Determine the output $Y_2(z)$ only in dependency of $V_1(z)$. (5 P)

$$\begin{aligned}
Y_3(z) =& V_2(z)H(z)z^{-1}\\
=& Y_1(z)H(z)z^{-1}\\
=& V_1(z)H(z)H(z)z^{-1}\\
=& V_1(z)H(z)^2z^{-1}\\
Y_2(z) =& V_3(z)H(z)z^{-1}\\
=& Y_3(z)H(z)z^{-1}\\
=& V_1(z)H(z)^2z^{-1}H(z)z^{-1}\\
=& V_1(z)H(z)^3z^{-2}
\end{aligned}$$

Hint:

$$\frac{1}{M}\sum_{k=0}^{M-1} \mathrm{e}^{jk2\pi n/M} = \begin{cases} 1, \text{for } k \bmod M \equiv 0 \\ 0, \text{else} \end{cases}$$