FACULTY OF ENGINEERING
CHRISTIAN-ALBRECHTS-UNIVERSITÄT
ZU KIEL

DIGITAL
SIGNAL PROCESSING AND
SYSTEM THEORY

Advanced Digital Signal Processing

Examination SS 2017

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Date:	14.09.2017
Name:	____________________________
Matriculation Number:	____________________________

Declaration of the candidate before the start of the examination

I hereby confirm that I am registered for, authorised to sit and eligible to take this examination.
I understand that the date for inspecting the examination will be announced by the EE&IT Examination Office, as soon as my provisional examination result has been published in the QIS portal. After the inspection date, I am able to request my final grade in the QIS portal. I am able to appeal against this examination procedure until the end of the period for academic appeals for the second examination period at the CAU. After this, my grade becomes final.

Signature: ____________________________

Marking

Problem	1	2	3
Points	/30	/25	/45

Total number of points: ________________ /100

Inspection/Return

I hereby confirm that I have acknowledged the marking of this examination and that I agree with the marking noted on this cover sheet.

☐ The examination papers will remain with me. Any later objection to the marking or grading is no longer possible.

Kiel, dated ________________ Signature: ____________________________

Advanced Digital Signal Processing

Examination SS 2017

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Date:	14.09.2017
Time:	09:00 h – 10:30 h (90 minutes)
Location:	KS2, C-SR I

Remarks

- Please check that you have received a cover sheet plus 4 sheets with 3 problems.
- Please write your **name** and your **matriculation number** on each sheet of paper that you return.
- Please keep your student ID and your identity card ready.
- During the exam only questions concerning the problems are answered.
- Please don't use any pencil or red pen.
- Please use a **new** sheet of paper with your name and matriculation number on it for **each problem**. You can ask for more sheets of paper, if necessary.
- The exam is open books, open notes; other people are closed. Programmable electronical devices except pocket calculators are not permitted.
- Partial credit will be given. No credit will be given if an answer appears with no supporting work or reason.
- Note that the given points of the subproblems are just preliminary.
- At the end of the exam put all sheets together as you have received them, including the problem sheets.
- No one is allowed to talk or to leave his or her seat until **all** exams have been collected.
- The problems and the solutions will be published on the website of the lecture. Also the date and the place of the inspection will be announced on this website.

Problem 1 (30 points)

This problem consists of four parts (a), (b), (c) and (d). They are **not** related to each other and can be solved **independently**.

(a) The period of a periodic discrete time signal is 0.25 milliseconds. Each period is sampled at 50 equally spaced points. It is assumed that with this number of samples, the sampling theorem is satisfied and thus there will be no aliasing.

(i) Compute the sampling period T. (1 P)

(ii) Compute the sampling frequency f_s. (1 P)

(b) A sinusoid signal $x(t)$ with a RMS-Amplitude of $10\text{V}/\sqrt{2}$ can be quantized with a 24-bit A/D converter containing a linear quantizer without clipping.

(i) Determine: The possible range R of the input signal for optimal A/D level control, the quantization levels L the quantizer has, and the optimal quantization step Δ. (2 P)

(ii) Sketch the mathematical model of the system with the added quantization noise after the sampling process $x(n) = x(n \cdot T)$. (1 P)

(iii) Compute the variance of the quantization error when the probability distribution of the quantization error can be assumed to be uniformly distributed and has zero mean. (2 P)

(c) Given is a sampled signal $x(n)$ that is time limited to 10 ms and has an essential bandwidth of 20 kHz.

(i) For the case where the number of signal points in the sampled version of the signal is equal to the number of FFT points ($L = M$) determine L the number of signal samples necessary to compute a radix-2 FFT with a frequency resolution (bin distance) of at least 50 Hz. (4 P)

(ii) What is the maximum sampling interval at which the signal must be sampled in order to retain the information present within the 20 kHz bandwidth. (2 P)

(iii) Does the signal need to be zero padded? If yes, for how much duration (in ms)? Explain your answer. (4 P)

(iii) Find the 4-point DFT $X(\mu)$ for an input signal $x(n) = \{1, 1, -1, 5\}$ and $\mu = 0 \dots 3$. (3 P)

(d) Find the response $y(n)$ of a causal system with the frequency response (10 P)

$$H(e^{j\Omega}) = \frac{e^{j\Omega} + 0.32}{e^{j2\Omega} + e^{j\Omega} + 0.16}$$

by using the Fourier transform to the input

$$x(n) = (-0.5)^n \delta_{-1}(n),$$

with

$$\delta_{-1}(n) = \begin{cases} 1, \text{ for } & n \geq 0, \\ 0, \text{ else.} \end{cases}$$

Problem 2 (25 points)

Given is a discrete signal $v(n) = \cos(\Omega_0 n)$. The corresponding continuous signal $v(t)$ was sampled with $f_s = \frac{1}{T_s} = 40$ kHz. $v(n)$ shall be decimated by a factor of $M = 2$. A anti aliasing filter $H(z)$ with

$$H(z) = \frac{1 + z^{-1}}{2}$$

is used before the signal gets down sampled.

(a) Determine and sketch the amplitude response $\left|H\left(e^{j\Omega}\right)\right|$ of the anti aliasing filter for $\Omega \in [0...2\pi]$. Label all axes. (8 P)

(b) Sketch the whole system with $V(\mathrm{e}^{j\Omega})$ being the input, $X(\mathrm{e}^{j\Omega})$ being the output of the anti aliasing filter and $Y(\mathrm{e}^{j\Omega})$ being the output of the down sampler. (3 P)

(c) Determine the amplitude spectrum of $X(e^{j\Omega})$ for $\Omega_0 = \frac{3\pi}{4}$. (2 P)

Now another signal

$$u(n) = 1 + \cos(\Omega_0 n) + \cos(\Omega_1 n)$$

with $\Omega_0 = \frac{3\pi}{4}$ and $\Omega_1 = \frac{\pi}{20}$is given to the system. The sample frequency used to sample the corresponding signal $u(t)$ is $f_s = \frac{1}{T_s} = 40$ kHz.

(d) What is the new sample frequency $f_{s,M}$ of $Y(e^{j\Omega})$? (2 P)

(e) Sketch the amplitude spectra $|X(e^{j\Omega})|$ and $|Y(e^{j\Omega})|$ for $\Omega \in [0...4\pi]$. Label all axis. (7 P)

(e) Explain which frequency components are existing in $|Y(e^{j\Omega})|$ and why that happend. (3 P)

Problem 3 **(45 points)**

This problem consists of two parts (a) and (b). They are **not** related to each other and can be solved **independently**.

(a) Consider the signal flow chart given in Figure 1 where $v(n)$ is the input of the system and $y(n)$ the corresponding output.

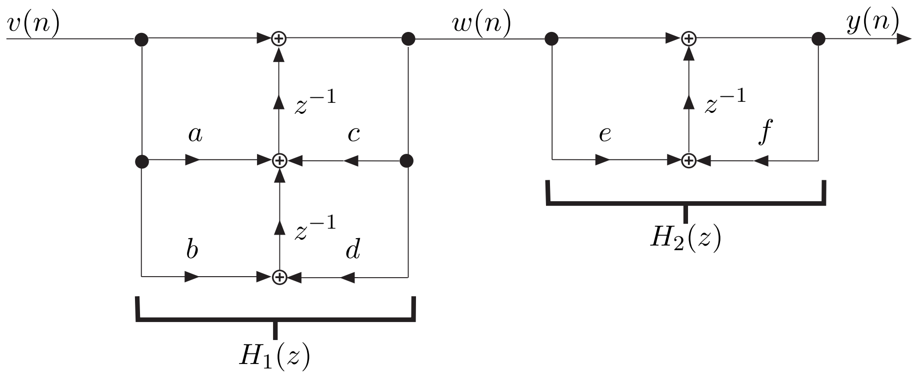

Figure 1: Signal flow chart.

(i) Of which structure is in the overall system? (1 P)

(ii) In which form are the systems $H_1(z)$ and $H_2(z)$ given? (1 P)

Suppose from now on the following coefficients for the system depicted in Figure 1.

$$a = \frac{4}{3} \qquad b = -\frac{4}{3} \qquad c = -\frac{1}{4} \qquad d = \frac{3}{8} \qquad e = -\frac{1}{2}$$

(iii) Choose the parameter f such that the system H_2 gets an allpass. (2 P)

(iv) Determine the transfer function $H_{tot}(z)$ of the overall system. (6 P)

(v) Derive the difference equation for $y(n)$ depending on $v(n)$. (3 P)

(vi) Determine the poles and zeros of the overall system and sketch the pole-zero diagram. (8 P)

(vii) Is the overall system stable? Give reason to your answer. (3 P)

(viii) What kind of filter is represented by the overall system (bandpass, lowpass, highpass, etc.)? Give reason to your answer. (3 P)

(b) A continuous lowpass filter with the following transfer function is given:

$$H_c(s) = \frac{1}{s^2 + 4s + 3}.$$

(i) This filter should be used as the basis for designing a digital filter $H_d(z)$. Which methods can be used to convert an analog filter design into a digital filter? (2 P)

(ii) Determine the transfer function $H_d(z)$ using the bilinear transform with $T_s = 1$. (6 P)

(iii) Which properties does the bilinear transform have? (3 P)

(iv) Why is the above described procedure of designing an analog filter and subsequent transformation to a digital filter useful? (2 P)

(v) Name two commonly used filter types that were treated in the lecture for designing lowpass filters. Which are the differences in the frequency response of these types? (5 P)