FACULTY OF ENGINEERING
CHRISTIAN-ALBRECHTS-UNIVERSITÄT
ZU KIEL

DIGITAL
SIGNAL PROCESSING AND
SYSTEM THEORY

Advanced Digital Signal Processing

Examination SS 2015

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Date:	08.09.2015
Name:	______________________________
Matriculation Number:	______________________________

Declaration of the candidate before the start of the examination

I hereby confirm that I am registered for, authorised to sit and eligible to take this examination.
I understand that the date for inspecting the examination will be announced by the EE&IT Examination Office, as soon as my provisional examination result has been published in the QIS portal. After the inspection date, I am able to request my final grade in the QIS portal. I am able to appeal against this examination procedure until the end of the period for academic appeals for the second examination period at the CAU. After this, my grade becomes final.

Signature: ______________________________

Marking

Problem	1	2	3
Points	/35	/36	/29

Total number of points: __________________ /100

Inspection/Return

I hereby confirm that I have acknowledged the marking of this examination and that I agree with the marking noted on this cover sheet.

☐ The examination papers will remain with me. Any later objection to the marking or grading is no longer possible.

Kiel, dated __________________ Signature: ______________________________

Advanced Digital Signal Processing

Examination SS 2015

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Date:	08.09.2015
Time:	09:00 h – 10:30 h (90 minutes)
Location:	C-SR I

Remarks

- Please check that you have received a cover sheet plus 5 sheets with 3 problems.
- Please write your **name** and your **matriculation number** on each sheet of paper that you return.
- Please keep your student ID and your identity card ready.
- During the exam only questions concerning the problems are answered.
- Please don't use any pencil or red pen.
- Please use a **new** sheet of paper with your name and matriculation number on it for **each problem**. You can ask for more sheets of paper, if necessary.
- The exam is open books, open notes; other people are closed. Programmable electronical devices except pocket calculators are not permitted.
- Partial credit will be given. No credit will be given if an answer appears with no supporting work or reason.
- Note that the given points of the subproblems are just preliminary.
- At the end of the exam put all sheets together as you have received them, including the problem sheets.
- No one is allowed to talk or to leave his or her seat until **all** exams have been collected.
- The problems and the solutions will be published on the website of the lecture. Also the date and the place of the inspection will be announced on this website.

Problem 1 (35 points)

Hint: The parts (a)-(d), (e)-(h) and (i)-(j) can be solved independently!

Given is a mixed-tone cosine signal $v_1(t) = \sum_{i=0}^{2} a_i \cos(2\pi f_i t)$ with the essential frequencies $f_0 = 1.0\,\text{kHz}$, $f_1 = 2.0\,\text{kHz}$, $f_2 = 4.0\,\text{kHz}$ and amplitudes $a_0 = 0.5\,\text{V}$, $a_1 = 0.25\,\text{V}$ and $a_2 = 0.75\,\text{V}$.

(a) Sketch the spectrum $V_1(j\omega) = \mathcal{F}\{v_1(t)\}$ of the signal $v_1(t)$. Label all axes! (4 P)

(b) To obtain an ideally sampled signal $v_{1,s1}(t)$, assume that signal $v_1(t)$ is multiplied with a periodic pulse train $p(t) = \sum_{n=-\infty}^{\infty} \delta_0(t - nT_s)$, with sampling frequency $f_{s1} = 1/T_s = 12\,\text{kHz}$. Sketch the spectrum $V_2(j\omega) = \mathcal{F}\{v_{1,s1}(t)\}$ from $-\omega_{s1}$ to ω_{s1}. (3 P)

(c) The sampling frequency is now changed to $f_{s2} = 1/T_s = 5\,\text{kHz}$ and in turn $v_{1,s2}(t)$ is obtained by multiplication of $v_1(t)$ with $p(t)$ (see part (b)). Sketch the spectrum $V_3(j\omega) = \mathcal{F}\{v_{1,s2}(t)\}$ from $-\omega_{s2}$ to ω_{s2}. (3 P)

(d) Which signal $v_{1,s1}(t)$ and/or $v_{1,s2}(t)$ could be correctly recovered by an ideal lowpass with the cutoff frequency $f_c = 5\,\text{kHz}$? (2 P)

In the following consider a sinusoid signal $v(n) = 0.5\text{V} \cdot \sin\left(\frac{\omega_0}{\omega_s} \cdot n\right)$ with $f_0 = 10\,\text{Hz}$ and $f_s = 10\,\text{kHz}$ which has to be quantized with a midrise quantizer. The word length of the quantizer is 3 bits. The quantizer has a digital full scale.

(e) Determine: The range R of the signal, the quantization levels L the quantizer has, and the quantization step Δ. (3 P)

(f) Sketch the input-output characteristic of the quantizer. Sketch all in subproblem (e) found parameters! (4 P)

(g) For time index n = 1200 calculate the quantized value $v_q(n)$ and the quantization error $e_q(n)$. (2 P)

(h) Sketch the real system and the mathematical model of the system with the added quantization noise. (2 P)

From now on consider a 4-point DFT of a filter impulse response $h(n)$ as

$$H(\mu) = \{2, -j, 0, j\}$$

and an input signal $x(n) = \{j, 2, -3, 1\}$.

(i) Find the 4-point DFT $X(\mu)$ for $x(n)$ for $\mu = 0 \ldots 3$. (5 P)

(j) Find the output signal $y(n) = x(n) \circledast h(n)$ by first finding $Y(\mu) = \text{DFT}\{y(n)\}$. (7 P)

Problem 2 (36 points)

Hint: The parts (a)-(d), (e)-(f) and (g)-(h) can be solved independently!

A system $H(z) = \frac{Y(z)}{V(z)}$ is given by the signal flow graph depicted in Figure 1.

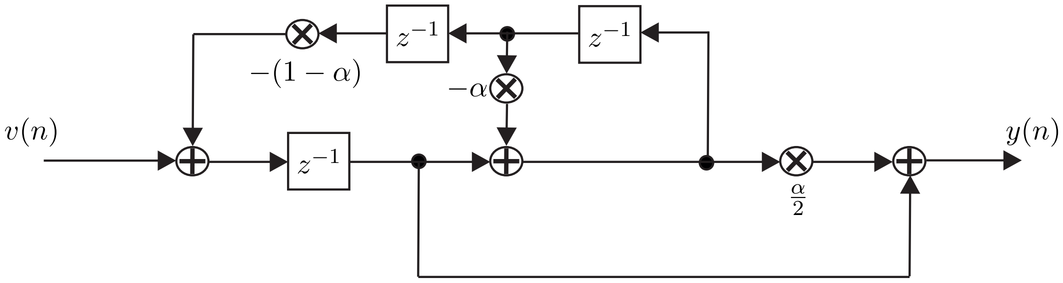

Figure 1: Signal flow graph of the system $H(z)$.

(a) Determine the system's transfer function $H(z) = \frac{Y(z)}{V(z)}$. Setup equations for each summation point. (10 P)

(b) Determine the difference equation of the system in dependency of $y(n)$ and $v(n)$. (2 P)

(c) Draw a Direct Form II realization, which realizes the same behaviour as the filter depicted in Figure 1. (5 P)

(d) Is the given filter stable for $\alpha = 0.5$? Give reasons for your answer. (2 P)

Hint: The first pole is at $z_{\infty,1} = -1$.

From now on the following transfer function is given:

$$H_1(z) = \frac{\left(\frac{2-\alpha}{1-\alpha}\right) z^2 - 1}{z^3 - z}.$$

The given system $H_1(z)$ has been determined by designing an analog system $H_\mathrm{a}(s)$ and transforming it to the digital domain by using the bilinear transform. The relationship between the s-plane and the z-plane is given by $s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$.

(e) Is the given filter stable? Give reasons for your answer. (5 P)

(f) Find the transfer function $H_\mathrm{a}(s)$ of the analog system, that could have been the basis for the design of the filter $H_1(z)$. Therefore assume a sampling interval $T = 2$ and $\alpha = 0.5$. (7 P)

Hint: Simplify the transfer function $H_\mathrm{a}(s)$ such that the numerator and denominator consist of one polynomial each.

In the following consider the linear-phase FIR filter impulse response with some unknown coefficients:

$$h_2(k) = \begin{cases} \{?,\ ?,\ 0,\ ?,\ ?,\ -0.3,\ -0.1\}, & 0 \le k \le 6, \\ 0, & \text{else}. \end{cases}$$

(g) Name all possible types of the linear-phase filter for $h_2(k)$. (2 P)

(h) Suppose now that $H_2(e^{j\pi}) = 0$ holds for the frequency response $H_2(e^{j\Omega})$ of $h_2(k)$. Furthermore, we require $\sum_k h_2^2(k) \leq 0.2$. Using this additional information, determine the final type of the linear-phase filter and complete $h_2(k)$. (3 P)

Problem 3 (29 points)

Hint: The parts (a)-(b), (c)-(d) and (e)-(m) can be solved independently!

Consider the multi-rate system depicted in Figure 2. The spectrum of $X(e^{j\Omega})$ is also depicted in Figure 2.

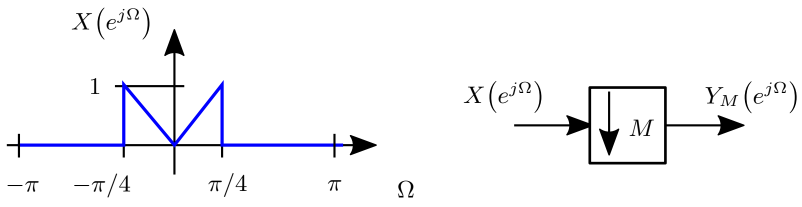

Figure 2: Block diagram of a multi-rate system.

(a) Sketch $Y_a(e^{j\Omega})$ for $M = 2, M = 4$, and $M = 8$ in the range of $-2\pi < \Omega < 2\pi$. For which case(s) does aliasing occur? (6 P)

(b) Is it possible to avoid aliasing? Explain **briefly** your answer. (1 P)

A non-integer downsampling by a factor of 7/3 can be performed by a multi-rate system.

(c) Explain **briefly** the necessary steps. (2 P)

(d) Sketch a block diagram. (2 P)

Consider the block diagram of a multi-rate system as depicted in Fig. 3.

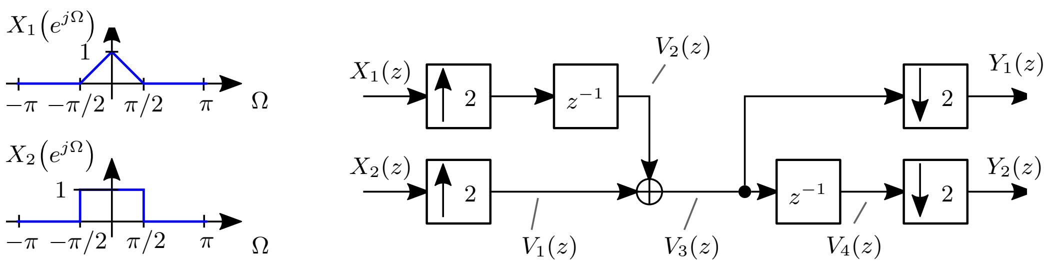

Figure 3: Block diagram of a multi-rate system.

(e) Give an expression for $V_1(z)$ in terms of $X_2(z)$. (1 P)

(f) Sketch $\left|V_1(e^{j\Omega})\right|$ in the range of $-2\pi < \Omega < 2\pi$. Label all axes! (2 P)

(g) Give an expression for $V_2(z)$ in terms of $X_1(z)$. (1 P)

(h) Sketch $\left|V_2(e^{j\Omega})\right|$ in the range of $-2\pi < \Omega < 2\pi$. Label all axes! (2 P)

(i) Give an expression for $V_3(z)$ in terms of $X_1(z)$ and $X_2(z)$. (1 P)

(j) Give an expression for $V_4(z)$ in terms of $X_1(z)$ and $X_2(z)$. Simplify as much as possible! (2 P)

(k) Give an expression for $Y_2(z)$ in terms $X_1(z)$ and $X_2(z)$. Simplify as much as possible! (3 P)

(l) Give an expression for $Y_1(z)$ in terms of $X_1(z)$ and $X_2(z)$. Simplify as much as possible! (3 P)

(m) Sketch a simplified block diagram of the system under consideration of the previous results. (3 P)