Digital Communications Project 8

Signal Sources and Spectral Analysis

1 Introduction

This lab is about determining the spectral content of a measured time-series signal. Studying the various methods for spectral analysis or spectral estimation is the goal of this lab.

Spectral analysis was first studied by Schuster in 1898 for detecting cyclic behavior in time-series. The word “spectrum” is coming from the latin word “specter”. Newton used the word “spectrum” to describe the decomposition of white light into various colors.

There are many applications of spectral analysis. In mechanical engineering the wear and tear of the mechanical parts like balls and bearings can be analyzed using measured signals. In speech signal processing, speech analysis and many speech enhancement techniques are based on the estimated speech spectrum. In radar and sonar signal processing the location of the sources are present in the spectral contents of the signal received. The electroencephalogram (EEG) signals measured from patients are spectrally analyzed to check for any symptoms.

2 Spectral Estimation

Intuitively, spectra of a signal can be thought of pieces of different frequencies arranged on a frequency scale. Hence in order to determine the power of the spectral content, the signal can be passed through a set of bandpass filters of desired bandwidth $\Delta(e^{j\Omega})$ and divide by the bandwidth to obtain an estimate power of the spectral estimate.

Let $y(n)$ is a *deterministic* discrete-time signal sequence with a finite energy

$$\sum_{n=-\infty}^{+\infty} |y(n)|^2 < \infty$$

then the *Energy Spectral Density*, $S(e^{j\Omega})$, of $y(n)$ is given by

$$S(e^{j\Omega}) = |Y(e^{j\Omega})|^2 \tag{1}$$

where $Y(\omega)$ is given by

$$Y(e^{j\Omega}) = \sum_{n=-\infty}^{+\infty} y(n)e^{-j\Omega n}. \tag{2}$$

Measured real-world signals are mostly random signals which are one of the many realizations of the random process from which these signals arise. The previous definition shown in Eq. (2) cannot be applied here for measured signals, even thought they don't change after measurement, because they do not have finite energy. Random signals are best described by their statistical properties like average power, second moment, etc. The spectral estimates of such signals are called as *Power Spectral Densities* or PSD given by

$$\phi(e^{j\Omega}) = \sum_{k=-\infty}^{\infty} r(k) e^{-j\Omega k} \tag{3}$$

where $r(k)$ is the autocovariance of the random signal $y(n)$ defined as

$$r(k) = E\{y(n) y^*(n-k)\}. \tag{4}$$

A second definition of the PSD, which is equivalent to Eq. (3), is given by

$$S(e^{j\Omega}) = \lim_{N\to\infty} E\left\{ \frac{1}{N} \left| \sum_{n=0}^{N-1} y(n) e^{-j\Omega n} \right|^2 \right\}. \tag{5}$$

2.1 Non-Parametric Estimation

Non-parametric methods for estimating spectra is based on the definitions introduced before. These methods introduced here are generic ways of estimating spectra and are not dependent on any models or parameters, hence the name non-parametric.

2.1.1 Periodogram Method

By modifying Eq. (5) the spectra of a random signal measurement can be obtained. This method is called as the Periodogram. The Periodogram is computed by

$$\hat{\phi}_{\mathrm{p}}(e^{j\Omega}) = \frac{1}{N} \left| \sum_{n=0}^{N-1} y(n) e^{-j\Omega n} \right|^2 . \tag{6}$$

Here the limit and the expectation has been omitted.

2.1.2 Corellogram Method

The correlation based computation of PSD is called as Corellogram. From Eq. (3) the Correlogram can be computed by

$$\hat{\phi}_{\mathrm{c}}(e^{j\Omega}) = \sum_{k=-(N-1)}^{N-1} \hat{r}(k) e^{-j\Omega k}. \tag{7}$$

An estimate of the covariance can be obtained by

$$\hat{r}(k) = \frac{1}{N-k} \sum_{n=k+1}^{N} y(n) y^*(n-k) \qquad 0 \leq k \leq N-1. \tag{8}$$

2.1.3 Blackman-Tuckey Method

To reduce high statistical variability a modified version of the Correllogram was developed called as the Blackman-Tuckey estimator. In this method a window $w(k)$ is applied on the estimated covariance. The window has a non-zero value within the window length M and zero outside it. The window is even function i.e. $w(-k) = w(k)$. Also $M < N$. It can also be applied to the Periodogram by convolving $W(e^{j\Omega})$, the DTFT of the window, with $\phi_\mathrm{p}(e^{j\Omega})$.

The Blackman-Tuckey estimator is obtained by

$$\hat{\phi}_\mathrm{BT}(e^{j\Omega}) = \sum_{k=-(M-1)}^{M-1} w(k) \hat{r}(k) e^{-j\Omega k}. \tag{9}$$

and

$$\hat{\phi}_\mathrm{BT}(e^{j\Omega}) = \hat{\phi}_\mathrm{p}(e^{j\Omega}) * W(e^{j\Omega}). \tag{10}$$

Different windows such as the Rectangular, Bartlett, Hann, Hamming can be used as per requirement.

2.1.4 Bartlett Method

One simple idea to reduce the variance in the estimated PDSs is to compute multiple PSDs from the given data length and average all of them. This is achieved by dividing the data into smaller segments of length M according to

$$y_i(n) = y(n + (i-1)L) \text{ for } i = 1, ..., L \tag{11}$$

so that $N = LM$ where N is the lenght of the measured signal. The Periodogram is then computed according to

$$\hat{\phi}_{i,\mathrm{p}}(e^{j\Omega}, i) = \frac{1}{M} \left| \sum_{n=0}^{M-1} y_i(n) e^{-j\Omega n} \right|^2 \text{ for } i = 1, ..., L \tag{12}$$

and the Bartlett estimate is obtained according to

$$\hat{\phi}_\mathrm{B}(e^{j\Omega}) = \frac{1}{L} \sum_{i=1}^{L} \hat{\phi}_{i,\mathrm{p}}(e^{j\Omega}, i). \tag{13}$$

2.2 Parametric Estimation

A second approach to estimating the spectra of measured random signals is to compare the signal with a model. The parameters estimated in the model are used for the estimation of the spectra. When the estimated parameters are close to the real parameters, then the estimated spectra is also more accurate. These methods in general are classified under parametric estimation.

In parametric modeling the measured signal $y(n)$ is thought of as a signal resulting from a white noise input to a system with transfer function $H(z)$. The white noise process has a covariance of $\sigma_{\text{wn}}^2\delta(k)$ and PSD as σ_{wn}^2. The transfer function $H(z)$ is described by

$$H(z) = \frac{b_0 + b_1 z^{-1} + \cdots + b_q z^{-q}}{a_0 + a_1 z^{-1} + \cdots + a_p z^{-p}}. \tag{14}$$

- **Autoregressive process** when $q = 0$. The filter is all-pole, recursive.
- **Moving Average process** when $p = 0$. The filter is all-zero, non-recursive.
- **ARMA process** when both $p, q > 0$. The filter is pole-zero, recursive.

The parameters that need to be estimated are the a_i for AR process, b_j for MA process and both a_i, b_j for ARMA process. The PSD of $y(n)$ is then given by

$$\phi(e^{j\Omega}) = |H(e^{j\Omega})|^2 \phi_{\text{wn}}(e^{j\Omega}) = \left| \frac{b_0 + b_1 z^{-1} + \cdots + b_q z^{-q}}{a_0 + a_1 z^{-1} + \cdots + a_p z^{-p}} \right|^2 \sigma_{\text{wn}}^2. \tag{15}$$

The parametric estimation is widely used in the analysis of speech signals. A simple all-pole filter can be assumed to describe the behavior of the vocal tract. Speech signals can be roughly divided into voiced and unvoiced segments. Unvoiced segments are those which are random noise like and voiced segments are those which are passed through a glottal pulse filter $G(z)$. The vocal tract filter can be modeled as

$$H(e^{j\Omega}) = \frac{1}{1 + \sum\limits_{i=1}^{p} a_i e^{-j\Omega i}}. \tag{16}$$

3 Lab Preparation

(a) Describe in a few words spectrum estimation.

(b) What is the difference between *Energy Spectral Density* and *Power Spectral Densities*?

(c) What is Non-Parametric estimation?

(d) What is Parametric estimation?

(e) List out the properties of speech signals.

4 Lab Execution

4.1 Periodogram and Corellogram

1. Realize a function called `periodg1` which computes an estimate of the PSD based on Eq. (6).
2. Load data from p1.mat. Use the above created function to estimate the PSD of the data.
3. Realize a function called `correlg1` which computes an estimate of the PSD based on Eq. (7).
4. Now use this function to compute the PSD of the above data.
5. Plot the above the two computed PSDs.
6. Repeat the above steps for p2.mat and p3.mat.
7. Plot all the estimations (using `subplot`).

4.2 Blackmann-Tuckey and Bartlett Estimation

1. Realize a function called `blackman1` which computes an estimate of the PSD based on Eq. (10). You could use `periodg1` and extend the same.
2. Load data from p1.mat. Use the above created function to estimate the PSD of the data.
3. Change the window used from rectangular to Barlett, Hann and Hamming.
4. Come up with a data sequence which would return the spectrum of the window used as PSD.
5. Plot the above spectrum for all windows stated above.
6. Realize a function called `bartlett1` which computes an estimate of the PSD based on Eq. (13).
7. Load data from p1.mat. Use the above created function to estimate the PSD of the data.
8. Try the above computations for p2.mat and p3.mat.
9. Plot all the estimations (using `subplot`).

4.3 Speech Signal Analysis

1. Load the data s1.mat and s2.mat.
2. Model the vocal tract with a AR process of order 10 using the `aryule` function.
3. Compute and plot the PSDs for both the data sets.
4. Try other values AR process order and plot the corresponding PSDs.
5. Based on the plots is it possible to determine voiced or unvoiced speech segments?